Offshoring and Domestic Labour Markets

A Matching Model of Outsourcing*

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Abstract

This paper studies offshore outsourcing by modelling it as a frictional process captured by the domestic firms' need to match with foreign producers. We highlight the effects of offshore outsourcing on wages, productivity and job flows, including their dynamics. We replicate four main robust features of the data with our model. First, outsourcing leads in the short run to closures and job destruction in the domestic market generating an increase in unemployment as has been observed during the nineties. Second, outsourcing leads to specialisation in high quality products in the home country as low productivity employment tends to be substituted by outsourcing. Third, the possibility of outsourcing raises the production possibilities of firms and raises the outside option to employment relationships. Fourth, if different tasks are complementary in the aggregate production, outsourcing leads to improvements in productivity in the medium term, increasing profits of firms, raising wages and after a period of higher unemployment ultimately increases employment.

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*The views expressed in this paper are those of the authors, and do not necessarily reflect the views of the European Central Bank or the Bank of Finland.

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1 Introduction

In the past decade the European labour market has undergone strong structural changes, one of the most prominent one stemming from the presence of international competition and global integration of production chains. Eastern European and Asian countries participate actively in the production chain of Western European firms and firms in Latin America provide an important base of production for companies loacted in the U.S.. Globalisation through reduced transportation and communication costs has made product markets more integrated, but production processes physically more distant. Baldwin (2006) coined it the great *unbundlings*. This international competition from low–wage countries has led to strong wage moderation for European and US workers and a rise in inequality. A large literature has focused on the effects of globalisation on worker's inequality with particularly detrimental effects on the income of unskilled workers due to an unfavourable skill premium and an increased unemployment risk (Krishna and Senses (2009)). Most recent developments also point to the fact that more and more activitiies of skilled workers are outsourced, such as software development or data processing as documented by Baldwin (2006), Blinder (2006) and Blinder (2009)).

Over the last two decades European labour markets have been characterised by first rising and then falling unemployment rates, before seeing an upsurge again recently. This has been attributed to rigid labour market institutions but also to the massive relocation of production sites outside Western Europe combined with an insufficient ability for fast restructuring. In addition, the inflation–unemployment link is today much weaker than a decade or two ago, captured by a flattening out of the Phillips curve. Reasons for this are multiple, but they overall reflect that the usual equilibrium forces between wage inflation and unemployment have weakened. Overall, domestic conditions, either on the side of the labour market or on the product market, no longer characterise total production conditions. Instead, international relationships and production possibilities, including trade and outsourcing affect wages, employment and domestic productivity.

The aim of this paper is twofold. First, we highlight the interrelation between domestic labour markets and a firm's decision to outsource specific parts of the production process. The firm's outsourcing possibility alters the behaviour of employment, wages and job flows, as expansionary and contractionary shocks to the economy also produce shifts in the international allocation of production. The integrated production structure including domestic labour and foreign production leads to a process of restructuring and ultimately to higher productivities due to cheaper production possibilities. Second, we model outsourcing as a frictional process, which to our knowledge is novel to the literature. We demonstrate and argue that the process of outsourcing requires a process of searching for and matching with foreign production partners, which appears analogous to the match formation in the domestic labour market. This process is characterized by uncertainties both for the matching as well as the breaking up process.

We replicate four main robust features of the data with our model. First, outsourcing leads in the short-run to job destruction in the domestic market generating an increase in unemployment as has been observed during the nineties. Second, outsourcing leads to a selection efect with higher productivities in the home country as low productivity employment tends to be substituted by outsourcing. This process is especially true for those tasks that are easily outsourceable at low cost. Third, the possibility of outsourcing raises the production possibilities of firms and raises the outside option to employment relationships. This reduces the joint surplus a domestic worker and firm generate in a match which, in turn, leads to initial wage moderation. Fourth, if different tasks are complementary in the aggregate production, outsourcing leads to improvements in productivity in the medium term, increasing profits of firms, raising wages and after a period of higher unemployment ultimately increases employment. The timing of medium–run effects depends on the degree of frictions in the labour market and the degree of complementarity between the different tasks.

To replicate these empirical facts the model includes frictional labour markets in the home country and frictional outsourcing possibilities. Domestic firms face a sequential decision of opening *positions*, interpretable as production planning, followed by an *allocation* decision to search in home and foreign markets for production inputs. A firm's value is hence composed by the number of employment relations and outsourcing contracts. The outsourcing possibility raises the productive capacity of the firm as it is no longer limited to the local labour market, at the same time wage dynamics are not confined to domestic labour market conditions, but are influenced by the offshoring possibilities.

The frictions for outsourcing find their rationale in the fact that domestic firms need to search for suitable business partners in the foreign country who have spare capacities and match the production requirements. Creating this outsourcing relationship involves search and costs. Once a productive outsourcing match is formed the production of the outsourced input may begin. This, however, involves costs of monitoring, communication and transportation which we capture for simplicity by a single parameter. The outsourcing relationships may be terminated for exogenous reasons. Indeed, business studies indicate that firms face a large amount of uncertainty when engaging in outsourcing (Deloitte 2005). We believe that the frictional process of outsourcing can capture a variety of these risks ranging from uncertainties in contract enforceability over exchange rate or transportation uncertainty to demand and supply uncertainty.

The domestic firm may face three types of destruction for jobs or outsourcing relationships. First, exogenous destruction for jobs or outsourcing relationships stemming from idiosyncratic factors, second, endogenous job destruction in the domestic labour market due to insufficient idiosyncratic productivity of the worker-employer match and, third, obsolescence of production processes due to restructuring or demand shifts. The destruction of domestic jobs or outsourcing relationships without obsolescence of the production process itself allows the firm to open new vacancies immediately. In the case of obsoles-

cence of the production process the firm is required to open in addition a new production position before searching for the necessary input factors. This modelling step is crucial for generating a tighter link between the home and foreign market for inputs.

In order to capture the most recent developments in the process of outsourcing, whereby also more skilled labour is outsourced in occupations that are easily outsourceable, the production process is composed of different occupations or tasks. These tasks may be outsourced with differing ease, modelled through differing communication costs for an effective integration of the outsourced production process into the domestic one. This implies that processes which are easily outsourceable tend to have a higher productivity cut-off level at which domestic production is unprofitable, thereby reducing domestic employment in this given task. Indeed, the general equilibrium effect of this leads to a reduction in vacancy opening for this task further magnifying the reduction in employment for the task.

Our model is most closely related to Mitra and Ranjan (2007) and Mitra and Ranjan (2009), but we focus on the dynamics of the outsourcing process in response to both temporary and permanent shocks. Felbermayr et al. (2008) introduce frictional labour markets into a canonical trade model with heterogeneous firms but do not focus on offshoring specifically. Our model includes shocks to the relative productivity of the home and domestic economy, shocks to communication and transportation costs and foreign labour (capacity) supply. The notion of trading tasks has been formalised by Grossman and Rossi-Hansberg (2008), we also build on their modelling style, but combine both a continuum of tasks and within each task different productivities, which allows to distinguish a skill related dimension (high vs. low skilled workers) and at the same time address the issue that some tasks are inherently more easily offshorable than others. This double dimension is key in analysing the macroeconomic and distributional implications of offshoring.

Other papers addressing similar issues include Zlate (2008), which focuses on the comovement of economic activity between the US and Mexico, stemming particularly from the offshoring of US production to the maquiladoras in the north of Mexico.

The structure of the paper is as follows. In section 3 we present a simplified version of the model, where both the labour market and the outsourcing market are modelled as a search-matching market with a single production task. We study different shocks in this benchmark model to demonstrate the forces at work. In section 4 we extend the model to incorporate two tasks with different degrees of offshorability, captured by differing communciation costs. We analyse transitions from the initial steady state of the economy to new ones in response to permanent changes in the costs of outsourcing. Furthermore we look at the adjustment to temporary shocks at business cycle frequencies. Section 5 concludes.

2 Empirical evidence

The empirical evidence on offshoring can be classified into three groups. The first one regards the extent of offshored jobs, i.e. the extent of the dislocation process, the second investigates the effects on employment, wages and productivity for remaining jobs in the domestic country, often with a distinction between high and low skilled workers, and the third group investigates the potential overall size of offshoring. We will discuss the different findings in turn and indicate how our model relates to these.

Little doubt exists that the global production pattern has been geographically fragmented. The question remains on the degree to which this has already taken place. For this the percentage of imports in intermediates is mainly used. This proxies the degree ny which companies in the home country have substituted intermediary steps of the production process in terms of goods and services by production abroad. This process requires that the different inputs to the final output are themselves self-contained and can be easily linked up with other production steps in the home country.

The OECD has calculated an index of offshoring for various countries using import ratios in materials and services. Table 1 presents the import ratio of materials and services in the respective sectors in 1995 and 2000. By this measure one can observe that offshoring has increased already during this period. It is worth noting that these figures understate the evolution during this decade, as offhoring has really taken off after the turn of the century.

On the effects of offshoring on employment and wages the overall findings appear very controversial. Geishecker and Görg (2007) find for German data that a percentage point increase of import share in intermediates reduces low-skilled worker wages by 1.5%, while it increases that of high-skilled workers by 2.6%. This clearly indicates the differing effects on different skill groups and puts a dividing line between high- and low-skilled workers. Consistent with these results Becker et al. (2009) use German evidence to show that offshoring reshapes their domestic workforce in favor of high-skilled workers. They find a positive correlation between offshoring and the firm's proportion of non-routine and interactive tasks, and find that offshoring predicts between 10 and 15 percent of observed changes in wage-bill shares of highly educated workers.

For the effects on employment two main channels are presumed in the theoretical literature, the downsizing effect and the productivity effect. Offshoring is mainly done for cost saving reasons by exploiting lower productivity adjusted wages abroad, which leads to a decline in employment for each preexisting domestic production site. At the same time, though, the higher productivity of the single firms allows to improve on the competitive advantage and increases the market share. Depending on which effect dominates, outsourcing can have positive or negative effects on employment, even in a partial equilibrium analysis. These effects have been found and isolated with German establishment data in Moser et al. (2009). Adding to this also the general equilibrium effect from free entry

	Panel A. Materials		Panel B. Services	
	1995	2000	1995	2000
Australia	3.7	3.9	2.2	2.1
Austria	8.3	10.5	2.9	3.8
Belgium	13.4	15.6	4.8	6.0
Canada	9.2	10.6	2.3	2.4
Denmark	7.6	7.5	2.4	2.8
Finland	9.5	11.0	1.5	2.3
France	5.6	5.8	1.2	1.1
Germany	6.0	8.1	1.2	1.9
Greece	5.2	4.7	1.4	1.7
Italy	7.0	7.7	1.6	2.0
Netherlands	11.5	11.9	3.8	4.4
Norway	6.2	5.7	2.9	6.4
Portugal	8.6	10.2	1.8	2.3
Spain	7.3	9.6	1.0	1.5
Sweden	9.1	10.6	3.2	4.0
United Kingdom	6.2	5.0	2.5	2.5
United States	2.4	2.8	0.5	0.7

Table 1: Offshoring in selected OECD countries, 1995 and 2000 (Source: OECD (2007))

and job creation is a daunting task from an empirical point of view. We try to capture these aggregate effects in this model and give an idea of the magnitudes by calibrating the model.

Regarding the third point, the the outlook for offshoring, Blinder (2006) initiated the debate on the future of the offshoring process and quantifies the share of today 's employment that might be offshorable. The two main conditions for offshorability are that the product or the service may be impersonally delivered and does not lose in quality if delivered over long distances. By analysing the detailed job characteristics of 817 occupations Blinder (2009) concludes that between 26 and 29% of US jobs are outsourceable. This does not imply that they are eventually outsourced. Table 2 offers an overview of the different estimates for offshoring by different authors. In our model we account for this dichotomous classification by attributing high or low communication costs to different tasks. If communication costs are infinite for the high-communication cost group, then we replicate the situation whereby a class of tasks is not offshorable, while the other class faces a threat of being offshored.

Authors	Jobs concerned	Countries concerned	Percent of tot. empl.	Year
Jensen and Kletzer	9.4 million	United States	9.4%	2000
Garner	14 million	United States	10%	2000
Van Welsum	23 million	United States	18.1%	2002
Beedham and Kroll	15 million	United States	11.7%	2003
McKinsey Global Inst.	160 million	World-wide	11%	2003
Blinder	$28-42$ million	United States	$20-25\%$	2004

Table 2: Estimates of service jobs that could potentially be moved offshore (Source: OECD (2007)/National Academy of Public Administration (2006))

3 Benchmark model with a single task

In this section we consider a model of outsourcing where domestic labour and internationally outsourced intermediate production is subject to search frictions. An important feature is that firms first need to open positions which may be used for domestic or outsourcing vacancies to search foreign production lines. The domestic labour and the foreign outsourcing market are specified in the search and matching setting following Pissarides (2000). Domestic employment is subject to exogenous endogenous job destruction whereas offshore outsourcing relationships are characterised only by exogenous destructions. The outsourcing relationship involves monitoring, communication and transportation costs captured by a single parameter. Vacanc

3.1 Matching in labour and outsourcing markets

3.1.1 Domestic labour market

The description of the domestic labour market follows closely Pissarides (2000). The law of motion for domestic employment is determined by the inflow of newly employed workers with idiosyncratic productivity going through the matching process and the outflow determined by job separations

$$n_t = \left(1 - \rho_t^{TOT}\right) \left[n_{t-1} + m\left(u_t, v_t\right)\right]. \tag{1}$$

The number of productive matches n_t is thus given by the number of productive matches in the previous period n_{t-1} and the number of new matches of the current period $m\left(u_t, v_t\right)$ that survive job destruction with total rate ρ_t^{TOT}. The number of matches is determined by a constant returns to scale matching function $m(u_t, v_t)$, increasing in the measure of searching unemployed workers $u_t = 1 - n_{t-1}$ and open vacancies v_t by firms. The probability of an unemployed worker finding a job is $s_t = m(u_t, v_t)/u_t$ and the probability for a firm to fill a vacancy $q_t = m(u_t, v_t)/v_t$, where s_t is increasing and q_t is decreasing in

market tightness $\theta_t = v_t/u_t$.

The job destruction rate consists of exogenous and endogenous separations of the employment relationship and an obsolescence rate of the position which is required to maintain employment or outsourcing relationships.

$$\rho_t^{TOT} = \rho^o + (1-\rho^o)\left[\rho^x + (1-\rho^x)\,\rho_t^n\right]. \tag{2}$$

In the case of obsolescence with rate ρ^O the worker enters the unemployment pool in the following period and the position disappears. Instead, following an exogenous ρ^x or endogenous destruction ρ_t^n job position remains open and may be reused by the firm. The endogenous job destruction rate is determined by the share of employed workers exhibiting an idiosyncratic productivity below a critical threshold $a_{it} \leq \tilde{a}_t$ and turn out to be non profitable

$$\rho_t^n = \Pr\left[a_t \leq \tilde{a}_t\right] = F\left(\tilde{a}_t\right), \tag{3}$$

where $F\left(a_{it}\right)$ is a time-invariant distribution of idiosyncratic productivities. The number of unemployed workers in a given period following the matching and destruction process is

$$\tilde{u}_t = 1 - n_t = 1 - \left(1-\rho_t^{TOT}\right)\left[n_{t-1} + m\left(u_t, v_t\right)\right] \tag{4}$$

where we have normalised the population size to 1.

3.1.2 Frictional outsourcing market

Search in the outsourcing market is analogous to search in the labour market. Firms create "vacancies" to form outsourcing relationships.[1] The law of motion for outsourcing relationships is hence

$$n_t^I = \left(1-\rho^I\right)\left[n_{t-1}^I + m\left(u_t^I, v_t^I\right)\right]. \tag{5}$$

The number of productive outsourcing matches n_t^I is thus given by their number in the previous period n_{t-1}^I and new matches $m\left(u_t^I, v_t^I\right)$ that survive job destruction ρ^I. v_t^I is the measure of "outsourcing vacancies" of domestic firms and u_t^I is the measure of idle foreign production capacity. The transition probability of foreign capacity being matched with a domestic firm and the transition probability of an open outsourcing vacancy getting filled are given by $s_t^I = m^I(u_t^I, v_t^I)/u_t^I$ and $q_t^I = m^I(u_t^I, v_t^I)/v_t^I$ respectively, where tightness in the outsourcing market is $\theta_t^I = v_t^I/u_t^I$. The job destruction rate consists of an exogenous separation rate an obsolescence rate of the position

$$\rho^I = \rho^o + (1-\rho^o)\,\rho^{xI}. \tag{6}$$

In the case of obsolescence the foreign production line enters the pool of idle capacity in the following period whereas the outsourcing position disappears. Upon exogenous destruction

[1] The "outsourcing vacancies" search for available capacity in the foreign country and the latter search for domestic firms. As an example one may think of a given number production lines suitable for automobile chassis production in China or Romania as capacity, and searching firms are domestic automobile producers.

both the outsourcing position and the foreign production line enter the matching pool. Foreign idle production capacity is the difference of total production capacity and the outsourcing relationships.

$$u_t^I = \chi_t^I - n_t^I. \tag{7}$$

Total production capacity χ_t^I in the outsourcing market can vary exogenously.

3.1.3 Vacancies

To open vacancies firms need to have positions available that can be used for employment vacancies and outsourcing vacancies

$$\tilde{x}_t = v_t + v_t^I. \tag{8}$$

The law of motion for positions depends on the number of existing positions, changes in employment and outsourcing relations and the active creation of new positions

$$\tilde{x}_t = (1-\rho^o)\left[\tilde{x}_{t-1} + \rho_t n_{t-1} + \rho^{xI} n_{t-1}^I - (1-\rho_t)\, q_t v_t - \left(1-\rho^{xI}\right) q_t^I v_t^I\right] + x_t. \tag{9}$$

x_t is the number of new job positions, $\tilde{x}_{t-1}$ are unmatched vacancies from the previous period, $\left(\rho_t n_{t-1} + \rho^{xI} n_{t-1}^I\right)$ are employment and outsouring destructions due to endogenous or exogenous factors which become available for new vacancies. The last two terms in square brackets are vacancies successfully matched which reduces the number of unfilled positions. All parts, except new positions are subject to an exogenous obsolescence rate ρ^O.

3.1.4 Households

Each household is thought of as a large extended family with a continuum of members on the unit interval. In equilibrium, some members are employed and others not: to avoid distributional issues, we assume that consumption is pooled inside the family, leading to perfect insurance among family members.[2] The representative household maximizes its lifetime utility from consumption C_t

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \tag{10}$$

Households own all firms in the economy and face the following per period budget constraint

$$C_t + \frac{B_t}{1+r_t} = w_t\left(\tilde{a}_t\right) n_t + (1-n_t)\, b + B_{t-1} + D_t - T_t, \tag{11}$$

where C_t is the quantity of aggregate consumption, B_t one-period bonds and r_t the real interest rate on the bonds. Income stems from earned wages $w_t\left(\tilde{a}_t\right) n_t$ and activity or benefits b of unemployed family members $(1-n_t)$ as interest rate payments and profits

[2] This is a common assumption in the literature; see e.g. Andolfatto (1996) and Merz (1995).

D_t from operating firms, net of government lump-sum taxes T_t used to finance unemployment benefits b. The household's decisions are also constrained by the law of motion for employment (1)

$$n_t = \left(1 - \rho_t^{TOT}\right)\left[n_{t-1} + s_t\left(1 - n_{t-1}\right)\right], \tag{12}$$

where s_t is the job-finding probability of a worker and taken as exogenous and u_t are searching workers at the beginning of the period. The Lagrangian for this problem is

$$\begin{aligned} \mathcal{L}^H &= \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} \right. \\ &\quad + \lambda_t \left[\mathcal{W}_t + \left(1 - n_t\right) b + B_t + D_t - T_t - C_t - \left(1 + r_{t-1}\right) B_{t-1}\right] \\ &\quad \left. + \mu_t^w \left[\left(1 - \rho_t^{TOT}\right)\left[s_t + n_{t-1}\left(1 - s_t\right)\right] - n_t\right] \right\}, \end{aligned} \tag{13}$$

where $\mathcal{W}_t = w_t\left(\tilde{a}_t\right) n_t$.Consumption maximization leads to the standard Euler condition

$$\lambda_t = \beta \mathbb{E}_t \left(1 + r_{t+1}\right) \lambda_{t+1} \tag{14}$$

$$\lambda_t = C_t^{-\sigma} \tag{15}$$

And the net value of employment compared to unemployment is

$$\frac{\mu_t^w}{\lambda_t} = \frac{\partial \mathcal{W}_t}{\partial n_t} - b + \mathbb{E}_t \left[\beta_{t+1}\left(1 - \rho_{t+1}^{TOT}\right)\left(1 - s_{t+1}\right)\frac{\mu_{t+1}^w}{\lambda_{t+1}}\right] \tag{16}$$

$$W_t - U_t = w_t\left(\tilde{a}_t\right) - b + \mathbb{E}_t\left[\beta_{t+1}\left(1 - \rho_{t+1}^{TOT}\right)\left(1 - s_{t+1}\right)\left(W_{t+1} - U_{t+1}\right)\right] \tag{17}$$

The value of employment net of unemployment $W_t - U_t$ consists of the wage income from an additional worker in the family $w_t\left(\tilde{a}_t\right)$ net of the value of unemployment b. To the contemporaneous value is added the discounted future value of employment conditional on surviving destruction.

3.2 The Firm's problem

3.2.1 Firm's objective function

Each period the domestic firms decide on the number of new positions to open and on the allocation of vacancies between the domestic labour market and the foreign outsourcing market to maximize the present discounted value of real profits. By producing domestically firms incur wage costs and vacancy posting costs. When outsourcing production, the costs consist of the price of the intermediate good, communication costs and the vacancy posting costs to find an unused production line. In both cases costs for vacancy creation are convex, similar to Fujita and Ramey (2007). In addition the firm pays a fixed cost for opening new job positions. The maximisation problem of a firm is as follows

$$\begin{aligned} \max_{\left\{n_t, n_{tt}^I, v_t, v_t^I, x_t, \tilde{a}_t\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta \left\{ n_t\left[A_t H_t\left(\tilde{a}_t\right) - w_t\left(a, \tilde{a}_t\right)\right] + n_t^I\left[A_t^I - \left(p_t + c_t\right)\right] \right. \\ \left. - \frac{\kappa}{2} v_t^2 - \frac{\kappa^I}{2}\left(v_t^I\right)^2 - \frac{1}{2} K x_t^2 \right\} \end{aligned} \tag{18}$$

subject to the laws of motion for labour and outsourcing relationships (1) and (5), the evolution of aggregate positions (8) and the law of motion of vacancies (9) .Domestic profits are generated from the output of domestic workers $n_t A_t H_t(\tilde{a}_t)$ net of their wage bill $w_t(a, \tilde{a}_t) n_t$, where $H_t(\tilde{a}_t) = \int_{\tilde{a}_t}^{\bar{a}} a \frac{dF(a)}{1-F(\tilde{a}_t)}$, and $w_t(\tilde{a}_t)$ is the median wage depending on the reservation productivity $\tilde{a}_t$.[3] Outsourcing profits arise from foreign output $n_t^I A_t^I$ net of the price of purchase p_t and communication costs c_t required to adequately coordinate the production process.

The first order necessary conditions are[4]

$$\partial n_t \quad : \quad \mu_t^J = A_t H_t(\tilde{a}_t) - w_t(a, \tilde{a}_t) + \beta \mathbb{E}_t (1-\rho^o) \left[\left(1-\rho_{t+1}\right) \mu_{t+1}^J + \rho_{t+1} \mu_{t+1}^x \right] \quad (19)$$

$$\partial n_t^I \quad : \quad \mu_t^I = A_t^I - (p_t + c_t) + \beta \mathbb{E}_t (1-\rho^o) \left[\left(1-\rho^{xI}\right) \mu_{t+1}^I + \rho^{xI} \mu_{t+1}^x \right] \quad (20)$$

$$\partial v_t \quad : \quad \mu_t^x = -\kappa v_t + (1-\rho^o)(1-\rho_t) q_t \left[\mu_t^J - \mu_t^x \right] + \beta \mathbb{E}_t (1-\rho^o) \mu_{t+1}^x \quad (21)$$

$$\partial v_t^I \quad : \quad \mu_t^x = -\kappa^I v_t^I + (1-\rho^o)\left(1-\rho^{xI}\right) q_t^I \left[\mu_t^I - \mu_t^x \right] + \beta \mathbb{E}_t (1-\rho^o) \mu_{t+1}^x \quad (22)$$

$$\partial x_t \quad : \quad K x_t = \mu_t^x \quad (23)$$

$$\partial \tilde{a}_t \quad : \quad (1-\rho^o) \frac{\partial \rho_t}{\partial \tilde{a}_t} (n_{t-1} + q_t v_t) \left(\mu_t^J - \mu_t^x \right) = n_t \left[A_t \frac{\partial H_t(\tilde{a}_t)}{\partial \tilde{a}_t} - \frac{\partial w_t(a, \tilde{a}_t)}{\partial \tilde{a}_t} \right] \quad (24)$$

The job values (19) and (20) consist of the current output net of the wage or foreign production costs and the continuation value. The continuation value is the respective match value in period $t+1$ conditional on the match not being destroyed (due to obsolescence, endogenous or exogenous destruction) and the value of a position in case of endogenous or exogenous destruction. The vacancy values (21) and (22) are given by the flow search cost in the respective market and the value of a filled vacancy if matching is successful and conditional on the match not being destroyed (due to obsolescence, endogenous or exogenous destruction). As positions return to the pool of open positions in case of endogenous or exogenous destruction, the firm obtains the value of an open position in case of endogenous or exogenous destruction. The value of an open vacancy depends on the value of a position and may fluctuate over time. The entry condition for positions is given by equation (23), linking the intensity of position creation to the value of a position. This is similar to the investments into capital as a pre-requesite to open vacancies as presented in Hornstein et al. (2007). Job positions that are opened remain in the labour market either as open vacancies or filled jobs until destroyed by obsolescence. If a job is destroyed due to exogenous job destruction, it reopens in the labour market as an open position. Finally, (24) determines the reservation productivity $\tilde{a}_t$ for endogenous job destruction.

[3] Wage determination will be discussed in more detail below.

[4] $\frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t} = \frac{f(\tilde{a}_t)}{1-F(\tilde{a}_t)} \left[\int_{\tilde{a}_t}^{\bar{a}} a \frac{f(a)}{1-F(\tilde{a}_t)} da - \tilde{a}_t \right] = \frac{f(\tilde{a}_t)}{1-F(\tilde{a}_t)} \left[H(\tilde{a}_t) - \tilde{a}_t \right]$
$\frac{\partial F(\tilde{a}_t)}{\partial \tilde{a}_t} = f(\tilde{a}_t)$

3.2.2 Wages

The median wage is determined by the Nash bargain between the firm and the median worker. We follow Hall and Milgrom (2008) by assuming that the outside value of the firm and worker is deferred wage negotiation as opposed to separating and starting to search for a new match. This implies that if the firm and worker agree on the wage they start production, whereas if they do not arrive to agreement, there will be no production this period and the continuation value is expected production next period.

The values of production and no production for the firm are given by

$$\mu_t^J = A_t H_t(\tilde{a}_t) - w_t(a, \tilde{a}_t) + \beta \mathbb{E}_t (1-\rho^o) \left[\left(1-\rho_{t+1}\right) \mu_{t+1}^J + \rho_{t+1} \mu_{t+1}^x \right] \quad (25)$$

$$\tilde{\mu}_t^J = 0 + \beta \mathbb{E}_t (1-\rho^o) \left[\left(1-\rho_{t+1}\right) \mu_{t+1}^J + \rho_{t+1} \mu_{t+1}^x \right] \quad (26)$$

and the surplus to the firm of reaching a wage agreement is given by

$$\mu_t^J - \tilde{\mu}_t^J = A_t H_t(\tilde{a}_t) - w_t(a, \tilde{a}_t). \quad (27)$$

For the worker the value of employment E_t in production and no production $\tilde{E}_t$ are given by

$$E_t = w_t(a, \tilde{a}_t) + \beta \mathbb{E}_t (1-\rho^o) \left[\left(1-\rho_{t+1}\right) E_{t+1} + \rho_{t+1} b \right] \quad (28)$$

$$\tilde{E}_t = b + \beta \mathbb{E}_t (1-\rho^o) \left[\left(1-\rho_{t+1}\right) E_{t+1} + \rho_{t+1} b \right] \quad (29)$$

where b is the value of leisure to the worker, and the surplus to the worker of reaching a wage agreement is given by

$$E_t^J - \tilde{E}_t^J = w_t(a, \tilde{a}_t) - b \quad (30)$$

The Nash bargain takes place between the values $\left(\mu_t^J - \tilde{\mu}_t^J\right)$ and $\left(E_t^J - \tilde{E}_t^J\right)$

$$\max_{w(a,\tilde{a}_t)} \left(\tilde{\mu}_t^J - \mu_t^J\right)^{(1-\eta)} \left(E_t^J - \tilde{E}_t^J\right)^{\eta} \quad (31)$$

where η is the bargaining power of the worker. The Nash first-order condition is

$$(1-\eta)\left[w_t(a, \tilde{a}_t) - b\right] = \eta \left[A_t H_t(\tilde{a}_t) - w_t(\tilde{a}_t)\right] \quad (32)$$

and produces the median wage

$$w_t(a, \tilde{a}_t) = \eta A_t H_t(\tilde{a}_t) + (1-\eta) b. \quad (33)$$

The median wage is a weighted average of match productivity and the value of leisure.

3.2.3 Job destruction

Finally, by using the wage equation and the law of motion for employment we can express the condition for endogenous job destruction as

$$\mu_t^J = (1-\eta) A_t \left[H(\tilde{a}_t) - \tilde{a}_t\right] + \mu_t^x \quad (34)$$

The value of a job is the share of the surplus going to the firm in addition to the entire value of an open position.

3.2.4 Resource constraint

Total output of the model is given by the output produced by both markets

$$Y_t = A_t H_t\left(\tilde{a}_t\right) n_t + A_t^I n_t^I. \tag{35}$$

The resource constraint of the economy is given by the assumption that the total output of the economy goes to the consumption of households and the position and vacancy costs of firms according to

$$Y_t = C_t + n_t^I\left(p_t + c_t\right) + \frac{\kappa}{2} v_t^2 + \frac{\kappa^I}{2}\left(v_t^I\right)^2 + \frac{1}{2} K x_t^2 \tag{36}$$

Value added in the home country is defined as production net of the payments done to the offshoring production plants ant the costs from frictions in the labour market and the offshoring activity.

$$VA_t = A_t H_t\left(\tilde{a}_t\right) n_t + n_t^I\left[A_t^I - \left(p_t + c_t\right)\right] - \frac{\kappa}{2} v_t^2 - \frac{\kappa^I}{2}\left(v_t^I\right)^2 - \frac{1}{2} K x_t^2. \tag{37}$$

3.3 Calibration

We linearise the respective equilibrium conditions around their deterministic steady state and then study the models properties by means of impulse responses to stochastic shocks and permanent deterministic shocks. The labour market parameters are calibrated to in line with the literature.[5] Direct evidence to calibrate the parameter values for the outsourcing market is scarcely available. We take the strategy of calibrating the outsourcing market to be similar to the domestic labour market in most respects. Thereafter we study the models properties by making deviations from the basic calibration of the outsourcing model.

The discount rate is set at $\beta = 0.99$ which implies an annual interest rate of 4%. The general productivity parameter is normalized to $\bar{A} = 1$ and the value of leisure is set to $b = 0.7$. The steady state unemployment rate is targeted to be $\bar{u} = 0.12$ and we determine the steady state employment rate $\bar{n}$ from the steady state equation for unemployment. The quarterly rate of filling vacancies is set to $\bar{q} = 0.71$, following den Haan et al. (2000). The matching function is standard Cobb-Douglas $m(u_t, v_t) = \gamma u_t^{\nu} v_t^{1-\nu}$, where γ is a scaling parameter and $\nu \in [0, 1]$ the elasticity of matches to the measure of unemployed workers.[6] We set $\nu = 0.5$ in accordance with empirical studies of the matching function. The bargaining power of workers is assumed to be $\eta = 0.5$ which is

[5] See eg. Walsh (2003, 2005), Trigari (2004), Krause and Lubik (2007) and den Haan et al. (2000), Christoffel et al. (2008).

[6] See e.g. Petrongolo and Pissarides (2001) and Blanchard and Diamond (1989). Petrongolo and Pissarides (2001) survey and estimate the different functional forms for matching functions. They can not reject a functional form with constant returns to scale such as the Cobb–Douglas form used here. For a detailed illustration of the matching model see Pissarides (2000).

conventional in the literature, although no direct evidence for this parameter is available. For the overall job destruction rate we use the conventional $\bar{\rho}^{TOT} = 0.1$ and for the obsolescence rate we use $\rho^o = 0.068$, a value similar to the rate of exogenous job destruction calibrated by den Haan et al. (2000) and the exogenous job destruction rate is set to $\rho^x = 0.01$. From the steady state job destruction condition (85) we obtain the steady state rate of endogenous job destruction $F\left(\bar{\tilde{a}}\right) = \frac{(\bar{\rho}-\rho^o)-(1-\rho^o)\rho^x}{(1-\rho^o)(1-\rho^x)} = 0.025$. We assume that a is uniformly distributed which implies $F(a) = \frac{a-\underline{a}}{\bar{a}-\underline{a}}$, $f(a) = \frac{1}{\bar{a}-\underline{a}}, H(\tilde{a}_t) = \frac{\bar{a}+\tilde{a}}{2}$ and $\frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t} = \frac{1}{2}$. With $a \in [0, 1.93]$ the steady state endogenous reservation productivity for job destruction is $\bar{\tilde{a}} = \underline{a} + F(a)(\bar{a} - \underline{a}) = 0.066$ and the expected productivity $\overline{H(\tilde{a}_t)} = 1.0$. Using the steady state equation for employment (86) and the value for $\bar{q}, \rho^o$ and $F\left(\bar{\tilde{a}}\right)$ we obtain the steady state measure of vacancies $\bar{v}$. With knowledge of $\bar{v}, \bar{u}$ and $\bar{q}$ we can determine the scaling parameter of the matching function γ from the steady state condition for the job finding rate $\gamma = \bar{q}\left(\frac{\bar{v}}{\bar{u}}\right)^{\nu}$.

In the outsourcing market we set the quarterly rate of filling vacancies equal to the vacancy filling rate in the domestic labour market $\bar{q}^I = \bar{q}$. We assume that the matching function in the foreign outsourcing market is symmetric to that of the domestic labour market so we have $\nu^I = \nu$ and $\gamma^I = \gamma$. The obsolescence rate ρ^o in the outsourcing market equals that in the domestic labour market and the overall destruction rate of outsourcing matches equals the overall job destruction rate in the domestic labour market $\rho^I = \bar{\rho}$. This implies an exogenous destruction rate $\rho^{xI} = 0.0343$ in the outsourcing market. We assume that the general productivity parameter is $\bar{A}^I = \bar{A}\overline{H\left(\tilde{a}\right)}$ so that matches in the labour market and in the outsourcing market are equally productive in the baseline calibration.

In the benchmark calibration we assume that the foreign outsourcing capacity $\bar{\chi}^I$ equals one in steady state. Consequently we can determine the measure of producing matches in the foreign market from the steady state equation for idle foreign production capacity (92). The steady state measure of vacancies in the outsourcing market $\bar{v}^I$ is determined by using the steady state equation for employment (87) and the values for $\bar{q}^I, \rho^o$ and ρ^{xI}. The steady state measure of open positions is given by the definition $\bar{\tilde{x}} = \bar{v} + \bar{v}^I$ and the steady state measure for new positions $\bar{x}$ is given by the steady state condition for positions (89).

Using the above values we can use the steady state job destruction condition (99) to determine the cost of opening a vacancy K. The search cost κ in the domestic labour market can then be determined from the steady state value of a vacancy in the domestic market (96), using the steady state value for a productive job given by (94). The foreign search cost is assumed to be symmetric to the domestic search cost $\kappa^I = \kappa$. The periodical cost of producing in an outsourcing relationship equals the price plus the communication cost $\bar{p} + \bar{c}$. This value can be determined from the steady state value of a filled position in the foreign outsourcing market (95). We assume that $\bar{p}$ and $\bar{c}$ are of equal size so they are simply given by half of the total cost.

The aggregate productivity shocks in both the home and the foreign country are assumed to follow an $AR(1)$ process with coefficient $\rho_A = \rho_A^I = 0.78$ and the standard

deviation of the shock is set to $\sigma_A = \sigma_A^I = 0.0088$.

Parameter	Value	Steady state	Value
Discount rate β	0.99	Unemployment rate $\bar{u}$	0.12
Value of leisure b	0.70	Idle capacity $\bar{\chi}$	0.12
Worker's bargaining power η	0.40	Vacancy filling rate $\bar{q}$	0.71
Elasticity of domestic matching ν	0.50	Foreign vacancy filling rate $\bar{q}^I$	0.71
Elasticity of foreign matching ν^I	0.50	Job destruction rate $\bar{\rho}^{TOT}$	0.10
Scale param. of dom matching γ	0.76	Reservation productivity $\tilde{a}$	0.066
Scale param. of foreign matching γ	0.76	Endog. job destruction $F(\tilde{a})$	0.025
Obsolescence rate ρ^o	0.068	Exp. idios. productivity $H(a)$	1
Exogenous destruction rate ρ^x	0.01	Employment rate $\bar{n}$	0.88
Foreign destruction rate ρ^{xI}	0.034	Filled outsourcing positions $\bar{n}^I$	0.88
Posting cost for new positions K	9.26	Vacancies $\bar{v}$	0.14
Search cost in labour market κ	1.45	Outsourcing vacancies $\bar{v}^I$	0.14
Search cost in foreign market κ^I	1.45	New positions $\bar{x}$	0.14
Aggregate productivity A	1.0	Positions $\overline{\tilde{x}}$	0.28
Foreign aggregate productivity A^I	1.0	Median wage $\bar{w}$	0.85
Agg. prod persistence ρ_A	0.95	Outsourcing costs $p + c$	0.85
Std. dev of prod shock σ_A	0.01		
Foreign agg. prod persistence ρ_A^I	0.95		
Std. dev. of prod shock σ_A^I	0.01		

Table 3: Parameters and Steady State Values in the Standard Model

3.4 Model analysis

In this section we present the main findings of the simple baseline model with one task to highlight the main driving forces that determine outsourcing decisions of firms and isolate the downsizing effect in the home country. The model below then integrates a continuum of tasks, distinguished by their communciation and transportation costs and discuss the role of task differences for employment dynamics. Here we first discuss the dynamic responses to business cycle variations in relative productivity. Second, we capture the permanent changes on communication costs induced by improvements in elecronic data transfers or transportation and on foreign production capacities in a deterministic model.

3.4.1 Temporary shocks in a stochastic model

Relative productivity (home/abroad) A productivity shock in the domestic economy boosts the output of domestic employees relative to the outsourced production lines,

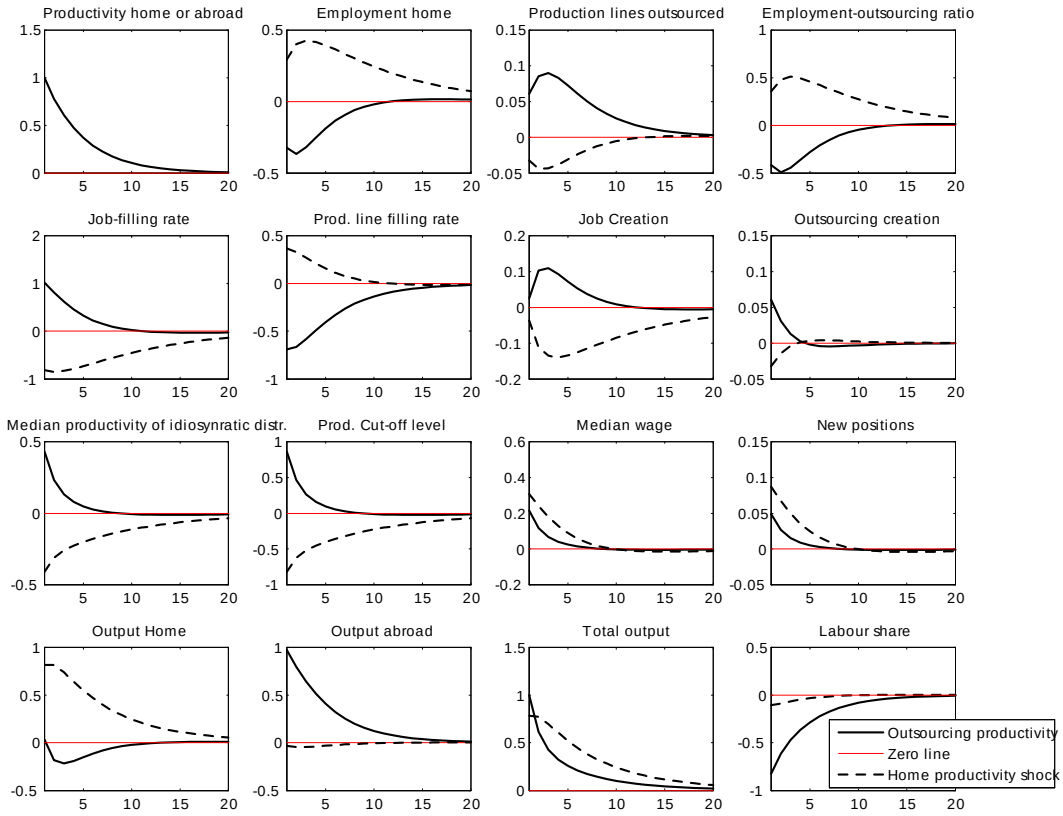

Figure 1: Equilibrium responses to a temporary productivity shock in the domestic economy and a temporary shock to outsourcing productivity.

this is captured by the dotted lines in Figure 1. Conversely, a positive shock to the outsourced production lines makes these more cost efficient than home employees and offshoring is intensified. The dynamics of the two shocks is not symmetric, though, due to differing job destruction types. Recall that home employment exhibits endogenous job destruction, which offers an additional adjustment margin to firms.

The main differences in the response to a 1% increase in the productivity of the two countries is that the effect for the region experiencing the shock are stronger than for the other region. A productivity increase in the home region affects more strongly employment than a productivity shock to the offshored production lines. In addition, the home productivity shock is characterised by larger persistence than the shock in the offshored region.

The transmission of the productivity shock works as follows. Higher home productivity leads to more new positions and vacancies directed towards the domestic market, but the decline in job-filling rate leads to a slight decline in job-creation. Nevertheless, employment in the home country increases due to a fall in endogenous job destruction which is

quantitatively more important. Indeed, the reservation productivity falls with the rise in aggregate productivity and consequently endogenous job destruction falls in the domestic labour market. As vacancy creation and employment shifts towards the domestic market, employment and output in the foreign market falls. Finally, the median wage in the home country increases due to higher productivity and the labour share only declines marginally due to the fact that the higher hiring reuqires adjsutment costs.

In the opposite case, in the presence of a positive productivity shock among the offshored production lines the increase in positions is channelled towards offshoring vacancies instead. While offshoring increases through mopre vacancies, the decline in employment is a combination of less vacancies and a higher productivity cut-off level, i.e. more endogenous job destruction. This is exactly the substitution of home employment with foreign production lines. Furthemore the labour share is strongly negatively affected due to less employment, but high value added that is generated and transformed into profits.

3.4.2 Permanent shocks in a deterministic model

As described by Blinder (2006) and Blinder (2009) in some detail, the profound change in the service industry for the future comes from technological advances in the area of data transmission and the capacity to transfer services over long distances without loss in quality. In parallel, productivity of emerging markets is also continuously improving due to catch-up effects, technological transfers and own innovative activity. Finally, the mere availability of a larger pool of workers due to an opening up of the world production structure has had strong influence on the production possibilities for firms in the developed countries. We capture these three effects, the communication cost channel, the porductivity increases and the extension of production capacities through three variables and discuss the impact of a permanent change to these.

Communication costs (technological improvements). The secular reduction in transportation and communication costs during the last decades has made product markets more integrated and simultaneously allowed production processes to be physically more distant Baldwin (2006). To capture the effects of technological improvements that allow for more distant production processes we introduce a permanent fall of communication costs into the model. It thereby becomes permanently cheaper to integrate the tasks perforemed in the foreign country into the overall production process. This, in turn, increases the incentives to offshore production either by opening new positions and allocating them into offshore production lines or instead by substituting existing employment relationships by offshored production. Output in the domestic and foreign country follow the evolution of domestic employment and foreign capacity utilisation. As may be seen in figure 2, new positions are indeed opened and job destruction is taking place in the home country. This increase in positions is transformed either in home vacancies or, in this case mainly in offshoring production lines, implying a steady increase in offshoring. Job creation

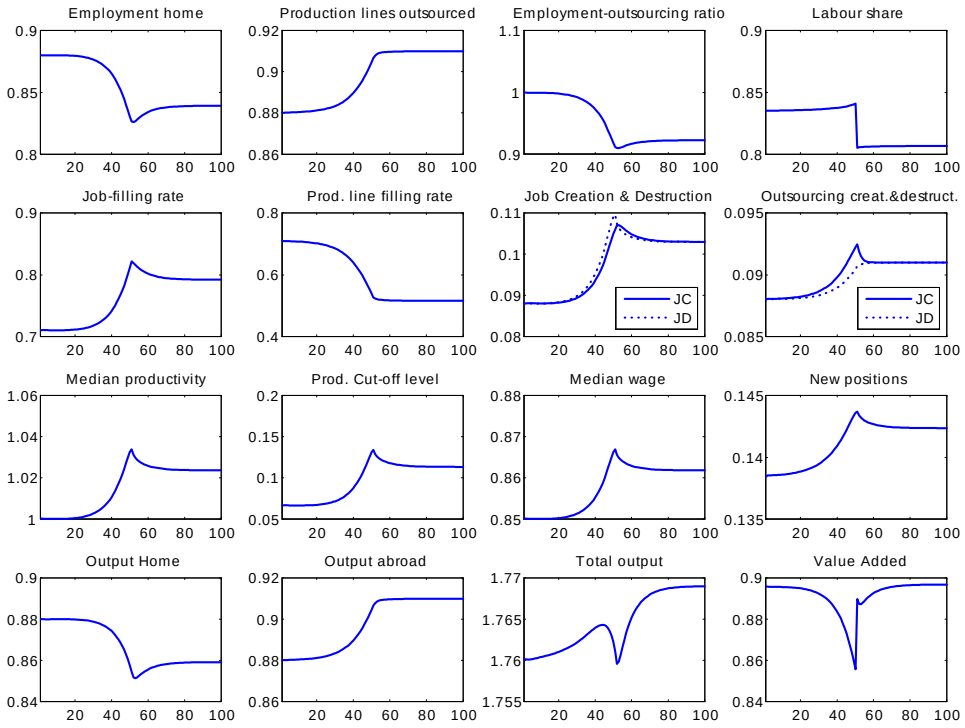

Figure 2: Permanent reduction in communication costs.

at home continues to take place in the home country, but job destruction is larger and peaks just before the initiatial fall in communication costs, leading to a fall in domestic employment and output. This reflects a higher turnover in the home country swith shorter employment spells. In the model employment hoarding takes place before the reduction in communciation costs in order to be easily transformed into offshoring production lines.

Increase in foreign labour input. Recent years have witnessed a spectacular increase in the global supply of labour, as Eastern European and Asian countries have increasingly been participating in the production chains of Western European and U.S. companies. It is a fully anticipated and permanent shock in labour supply. This expansion of global labor supply provides firms with the opportunity to benefit from low cost foreign labour.

To study the effect of the increase in labour supply we introduce a permanent but gradual (over a period of five years) rise of foreign production capacity χ into the model. The main adjustment channel is through the ease of finding new production possibilities in the foreign market. The increased number of production lines abroad increases the probability of finding a foreign business partner for a European firm, hence the value of engaging in search increases and domestic firms open more vacancies abroad to establish production lines abroad. Production lines and output abroad increase as expected, and with a lag

due to matching frictions. This requires further adjustments on two channels, first, more positions need to be created from which a larger share tend to lead to outsourcing, and second a substitution takes place away from domestic labour towards outsourcing through the domestic job destruction channel. The cut-off productivity for domestic jobs increases and more low productivity jobs are destroyed. Having a low idiosyncratic productivity realization is less worth keeping, because the prospects in the foreign market increase.

At the moment of the expansion of production sites abroad it is very easy to find a new business partner and creation of offshored production lines peaks. Contemporaneously, existsing employment relationships are transformed into offshoring and require replacement. This takes place through an initial peak in vacancies in the domestic market due to a negative congestion externality and at the same time due to higer job destruction levels more churning takes place domestically. Domestic employment falls because the increase in the cut-off level for the idiosyncratic productivity is generating more job destruction than jobs are created.

The gradual increase in the foreign labour supply has as consequence that the labour share in domestic value added continuously declines. This is the case although value added itself declines due to the fact that more offshoring takes but retained profits in teh home country are smallesr. Totaloutput, instead is continuously increasing.

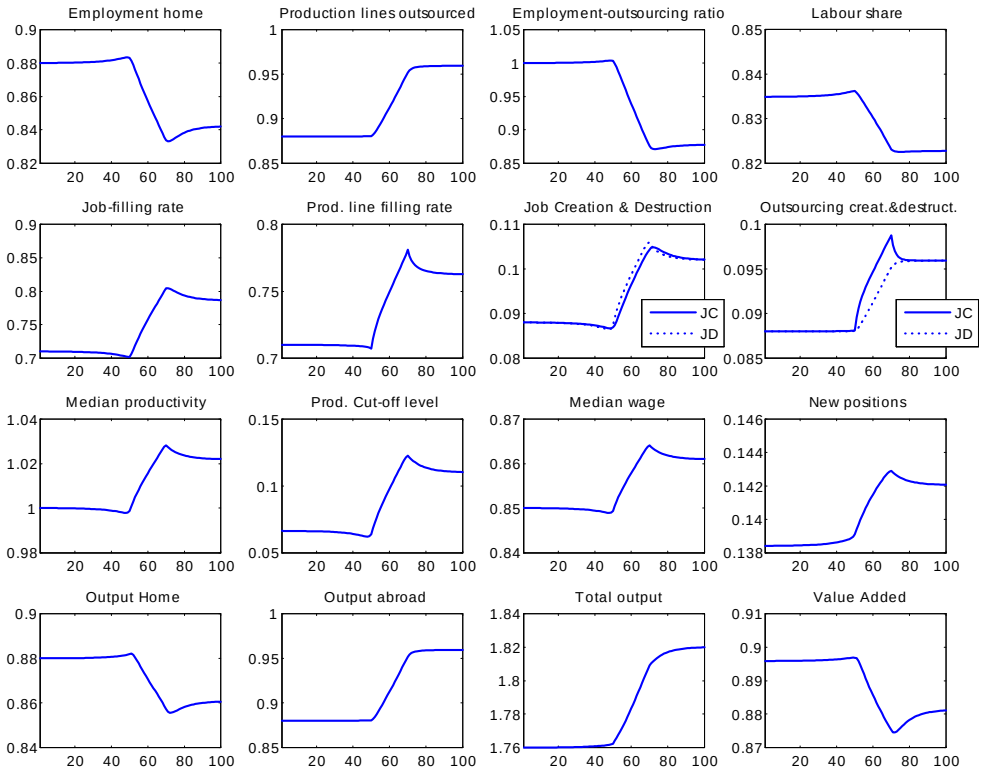

Figure 3: Gradual increase of foreign labour input in 5 year period by 10 percent.

4 Model with different degrees of offshorability

The single-task model in the former section highlighted the main mechanisms of job-destruction and creation abroad and at home with consequences on employment, productivity, output, value added etc. This section address more specifically the issue of changing degrees of offshorability between types of tasks. The technological advancements in communication technologies ultimately affect the decision on where to locate production plants and where to produce services before transferring them to other locations. Blinder (2006) emphasised that tasks can be broadly divided into two classes, those that are offshorable and those that need physical contact for their delivery. We model the degree of offshorability by the communication costs that are incurred when outsourcing. The continuum of tasks are split into the two groups of either high or low communication costs, the shares of which are determined by a parameter π. Once this distinction is made the relative outsourcing costs determine to what degree outsourcing indeed takes place.

4.1 The model

4.1.1 Matching and flows

A continuum of tasks exist in the economy that can be grouped into those with high outsourcing costs (H) and those with low outsourcing costs (L). When being opened positions draw a high or a low communication cost with probability π^H and $1-\pi^H$ respectively. The positions can then be used for opening home vacancies v_t^i or outsourcing vacancies v_t^{Ii} in the specific task. The process of matching vacancies with either employees or foreign unused production lines takes place through matching functions that combine both types of tasks. Search is thus undirected. Job filling rates in the foreign and domestic markets are then given by

$$q_t = q^{param}\left(\frac{\sum_{i=L,H} v_t^i}{u_t}\right)^{-\gamma} = q^{param}\left(\frac{v_t^L + v_t^H}{1-n_t^L-n_t^H}\right)^{-\gamma} \tag{38}$$

$$q_t^I = q^{param}\left(\frac{\sum_{i=L,H} v_t^{Ii}}{u_t^I}\right)^{-\gamma} = q^{param}\left(\frac{v_t^{IL} + v_t^{IH}}{1-n_t^{IL}-n_t^{IH}}\right)^{-\gamma} \tag{39}$$

As in the simple model above positions in both the foreign and domestic market are destroyed by obsolescence at rate ρ^o. For job matches surviving obsolescence, destruction may take place due to exogenous and endogenous separations at the rate ρ_t^i, given by

$$\rho_t^i = \rho^x + (1-\rho^x)\, F\left(\tilde{a}_t^i\right), \quad i = L, H. \tag{40}$$

In the foreign market matches that survive obsolescence may be destroyed only exogenously at rate ρ^{xI}. The overall separation rates in the domestic an foreign market are hence

$$\begin{aligned} \rho_t^{TOT} &= \rho^o + (1-\rho^o)\,\rho_t^i, \quad i = L, H \\ \rho^I &= \rho^o + (1-\rho^o)\,\rho^{xI}. \end{aligned}$$

The evolution of employment in the domestic and foreign market are given by

$$n_t^i = (1-\rho^o)\left(1-\rho_t^i\right)\left(n_{t-1}^i + q_t v_t^i\right), \quad i = L, H \tag{41}$$

$$n_t^{Ii} = (1-\rho^o)\left(1-\rho^{xI}\right)\left(n_{t-1}^{Ii} + q_t^I v_t^{Ii}\right), \quad i = L, H. \tag{42}$$

Vacancies in the domestic labour market and the foreign outsourcing market are created through the opening of positions. Matched vacancies leave the pool of positions into production of the respective type, but return to the respective pool upon exogenous or endogenous separations. All except new opened positions are subject to the obsolescence shock. Following the earlier notation the measure of new positions x_t is

$$x_t = x_t^L + x_t^H \tag{43}$$

and the total measure of positions of type i is given by

$$\tilde{x}_t^i = v_t^i + v_t^{Ii} \tag{44}$$

and the law of motion for positions of type i is

$$\begin{aligned} \tilde{x}_t^i &= (1-\rho^o)\{\tilde{x}_{t-1}^i \\ &\quad -\left(1-\rho_t^i\right) q_t v_t^i - \left(1-\rho^{xI}\right) q_t^I v_t^{Ii} \\ &\quad +\rho_t^i n_{t-1}^i + \rho^{xI} n_{t-1}^{Ii}\} + \pi^i x_t, \quad i = L, H \end{aligned} \tag{45}$$

4.2 The Firm's problem

4.2.1 Final goods firms

The production function for the final good combines the continuum of tasks using a Dixit-Stiglitz aggregator with a degree of substitution σ between the individual tasks

$$Y_t = \left[\int_0^1 \alpha(\tau)^{\frac{1}{\sigma}} y_t(\tau)^{\frac{\sigma-1}{\sigma}} d\tau\right]^{\frac{\sigma}{\sigma-1}}, \tag{46}$$

where $y_t(\tau)$ is the amount of task τ inputs in order to produce the homogenous final output good Y_t. The parameter $\alpha(\tau)$ can be interpreted as a productivity or preference parameter. The cost minimisation problem leads to the demand function for each task

$$y_t(\tau) = \alpha(\tau)\left(\frac{p_t(\tau)}{P_t}\right)^{-\sigma} Y_t, \tag{47}$$

where $P_t = \left[\int_0^1 \alpha(\tau) p(\tau)^{1-\sigma} d\tau\right]^{\frac{1}{1-\sigma}}$ is the aggregate price index for purchasing one unit of final good, normalized to 1 subsequently. Each intermediate task τ is produced by a large representative firm with jobs of differing productivities. Prices for the different intermediate tasks are determined in competitive markets. With the normalisation of the aggregate price, the relative prices of intermediate tasks becomes:

$$p_t(\tau) = \left(\frac{y_t(\tau)}{\alpha(\tau) Y_t}\right)^{-\frac{1}{\sigma}} \tag{48}$$

Considering mainly two types of communication costs, high and low, it is possible to rewrite the production function as:

$$Y_t = \left[\pi^H \alpha(h)^{\frac{1}{\sigma}} y_t(h)^{\frac{\sigma-1}{\sigma}} + \left(1-\pi^l\right)\alpha(l)^{\frac{1}{\sigma}} y_t(l)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

and the price level is $P_t = \left[\pi^H \alpha(h) p(h)^{1-\sigma} + \left(1-\pi^l\right)\alpha(l) p(l)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$.

$$\left(\frac{y_t(\tau)}{\alpha(\tau) Y_t}\right)^{\sigma} P_t^{-\sigma} = p_t(\tau)^{-\sigma}, \tag{49}$$

$$P_t = \left[\pi^H \alpha(h)\, p(h)^{1-\sigma} + \left(1-\pi^l\right)\alpha(l)\, p(l)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{50}$$

From (48) follows the relative size of inputs of tasks determined by the relative price, the productivity $\alpha(\tau)$ and the elasticity of substitution:

$$\frac{p_t(l)^\sigma y_t(l)}{\alpha(l)} = Y_t = \frac{p_t(h)^\sigma y_t(h)}{\alpha(h)} \tag{51}$$

$$\frac{y_t(l)\,\alpha(h)}{\alpha(l)\, y_t(h)} = \left(\frac{p_t(h)}{p_t(l)}\right)^\sigma. \tag{52}$$

4.2.2 The maximisation problem

Heterogenity across firms exists across tasks in form of communication costs and for each task across productivities. We aggregate firms to a representative firm of a continuum of tasks and high and low communication costs.

$$\max_{\substack{\{n_t^L, n_t^H, n_t^{IL}, \\ n_t^{IH}, v_t^L, v_t^H, v_t^{IL}, \\ v_t^{IH}, x_t, \tilde{a}_t\}_{t=0}^\infty}} E_0 \sum_{t=0}^{\infty} \beta \left\{ \sum_{i=\{L,H\}} \pi^i \left[n_t^i \left[p_t^i A_t H_t\left(\tilde{a}_t^i\right) - w_t^i\left(\tilde{a}_t^i\right)\right] + n_t^{Ii}\left[p_t^i A_t^I - \left(p_t + c_t^i\right)\right] \right.\right.$$
$$\left.\left. -\frac{\kappa}{2}\left(v_t^i\right)^2 - \frac{\kappa^I}{2}\left(v_t^{Ii}\right)^2\right] - \frac{K}{2}x_t^2 \right\} \tag{53}$$

$$s.t.\ \text{equations}\ (41),\ (42),\ (43),\ (44),\ (45) \tag{54}$$

Firms maximize profits for the shares of high and low communication cost tasks subject to the laws of motion for employment, production lines, positions and the equalities between total vacancies andthe number of total positions for each type of task, L and H. The firm's problem has an analogous interpretation to that in the simple model above, except that now the firm operates with a contiuum of tasks grouped into two groups. The first order conditions are

$$\partial n_t^i : \mu_t^{Ji} = p_t^i A_t H_t\left(\tilde{a}_t^i\right) - w\left(\tilde{a}_t^i\right) + (1-\rho^o)\,\beta\mathbb{E}_t\left[\left(1-\rho_{t+1}^i\right)\mu_{t+1}^{Ji} + \rho_{t+1}^i \mu_{t+1}^{xi}\right] \tag{55}$$

$$\partial n_t^{Ii} : \mu_t^{Ii} = p_t^i A_t^I - \left(p_t + c_t^i\right) + (1-\rho^o)\,\beta\mathbb{E}_t\left[\left(1-\rho^{xI}\right)\mu_{t+1}^{Ii} + \rho^{xI}\mu_{t+1}^{xi}\right] \tag{56}$$

$$\partial v_t^i : \mu_t^{xi} = -\kappa v_t^i + (1-\rho^o)\left(1-\rho_t^i\right) q_t\left[\mu_t^{Ji} - \mu_t^{xi}\right] + (1-\rho^o)\,\beta\mathbb{E}_t\mu_{t+1}^{xi} \tag{57}$$

$$\partial v_t^{Ii} : \mu_t^{xi} = -\kappa^I v_t^{Ii} + (1-\rho^o)\left(1-\rho^{xI}\right) q_t^I\left[\mu_t^{Ii} - \mu_t^{xi}\right] + (1-\rho^o)\,\beta\mathbb{E}_t\mu_{t+1}^{xi} \tag{58}$$

$$\partial x_t : Kx_t = \sum_{i=L,H} \pi^i \mu_t^{xi} = \pi^H \mu^{xH} + \left(1-\pi^H\right)\mu^{xL} \tag{59}$$

$$\partial \tilde{a}_t : (1-\rho^o)\frac{\partial \rho_t^i}{\partial \tilde{a}_t^i}\left(n_{t-1}^i + q_t^i v_t\right)\left(\mu_t^{Ji} - \mu_t^{xi}\right) = n_t^i \frac{\partial}{\partial \tilde{a}_t^i}\left[p_t^i A_t H_t\left(\tilde{a}_t^i\right) - w_t\left(\tilde{a}_t^i\right)\right] \tag{60}$$

with a similar interpretation to the first order conditions of the model with a single task. It is instructive to notice that from equations (57) and (58) we obtain an arbitrage condition between home production and outsourcing

$$\kappa^I v^{Ii} - \kappa v^i = (1-\rho^o)\left\{\left(1-\rho^{xI}\right) q_t^I \left[\mu_t^{Ii} - \mu_t^{xi}\right] - \left(1-\rho_t^i\right) q_t \left[\mu_t^{Ji} - \mu_t^{xi}\right]\right\}. \tag{61}$$

This equation states that the difference in the search costs in the foreign and domestic market equals the difference in expected payoffs of search in the two markets. The latter depends on the value of production as well as market tightness in each market.

4.2.3 Wage

The median wage for a domestic job match of type $i = H, L$ is derived as above from the values of production and the value of deferred bargaining for both the firm and the worker. The Nash bargaining problem produces

$$w\left(\tilde{a}_t^i\right) = \eta p_t^i A_t H\left(\tilde{a}_t^i\right) + (1-\eta)\, b \tag{62}$$

for a match of type i.The wage eqauation implies a wage differntial between high and low communication cost tasks affected by the endogenous cut-off level for productivity $\tilde{a}_t^i$ and the relative price p_t^i.

4.2.4 Job destruction

From the first order condition for the reservation productivity $\tilde{a}_t^i$ of a domestic match of type i, and substituting the wage equation (62) and the law of motion for employment (41) , we obtain

$$\begin{aligned} \mu_t^{Ji} &= (1-\eta)\left(1-\rho^x\right) p_t^i A_t \left[H\left(\tilde{a}_t^i\right) - \tilde{a}_t^i\right] + \mu_t^{xi} && (63) \\ &= (1-\eta)\left(1-\rho^x\right) p_t^i A_t \left[H\left(\tilde{a}_t^i\right) - \tilde{a}_t^i\right] + \left(K x_t - \pi^{-i}\mu^{-i}\right)/\pi^i && (64) \end{aligned}$$

where the superscript $-i$ refers to the other group of communication costs. The value of a job is the share of the surplus going to the firm in addition to the entire value of the position. In addition, the higher is the current valuation of positions, the higher the surplus. This is the case the more positions are opened in general (Opening positions benefits the valuation of both types of tasks due to the exogenous probabilities π^i.

4.3 Calibration

In the analysis of the model with differing degrees of outsourceability we follow the baseline calibration provided in section 3.3. In addition we set the probabilitiy of drawing a high communication cost to $\pi^H = 0.5$. As $\pi^L = 1 - \pi^H$ we have $\pi^L = 0.5$.

In our initial calibration the (H) and (L) tasks are identical in terms of outsourcing costs before a shock takes place. This implies that the tasks are *de facto* symmetric in terms of all variables and the intial steady state of the model with differing degrees of outsourceability is identical that of the simple model in section 3. Only after the outsourcing cost is shocked, the two tasks differ in outsourceablity.

4.4 Model analysis

4.4.1 Temporary shocks in a stochastic model

[t.b.c]

4.4.2 Permanent shocks in a deterministic model

We now proceed to study unanticipated permanent shocks in a deterministic model. As with the simple model we start by considering a permanent fall in communication costs, as this is one of the shocks that have featured prominently in recent studies. We then proceed to other shocks.

Communication costs (technological improvements) Similar to the simple model setup above we consider a permanent fall of communication costs of 10% in a single period, but now only for the low cost task. This is done to analyse the effects of the secular reduction in transportation and communication costs during the last decades which affected a subset of tasks, as some are not outsourceable by their characteristics. The implication of this fall in communciation costs is that the low cost task becomes permanently cheaper to operate in the foreign country. To take advantage of this firms have two margins, either they open up new positions, which are either of the high or low type task and allocate them through vacancies into offshored production lines. Alternatively they substitute existing employment relationships with new offshored relationships. Indeed, as a share $1 - \pi^H$ of new positions get the low cost draw the expected value of a position increases, which leads to a rise in the opening of new and total positions. As the shares π^H and $1 - \pi^H$ of high and low cost positions are drawn randomly, also the number of high cost vacancies increases.

The new cost structure leads to a reduction of vacancy allocation of the low cost task in the domestic labour market and an increase in vacancies in the low cost task to the foreign country. The opposite happens for high cost tasks. The exogenous allocation of new positions to high and low cost tasks makes that more high cost vacancies are opened, which themselves are more easily filled due to a single matching function. The fact that low cost tasks are outsourced increases the vacancy filling rate from which vacancies of high cost (non-offshorable) tasks profit. These get easily filled and employment increases for these tasks in the domestic labour market. In addition job destruction takes place in the home country, which captures the substitution or downsizing effect, by which firms

substitute home employment with offshored production. The productivity cut-off level for the low-cost tasks increases, i.e. endogenous job-destruction increases. This is due to the lower costs of offshored production. The overshooting taks place because the outside option for such an employment relationship is offshoring, its value being determined by the intensity with which new positions are indeed created. When positions become more valuable, firms endogenously destroy more domestic low cost jobs and the cut-off level increases.

The evolution of employment and foreign production lines in operation follows the evolution of domestic labour market vacancies (and job destruction) and the evolution of foreign outsourcing vacancies with a lag as search and matching takes time. Output, in turn, follows domestic employment and foreign production lines. In the aggregate, total output increases due to lower production costs abroad and the endogenous repsonse towards shifting production offshore for these types of tasks. Frictional labour markets with endogenous job destruction implies that the transition in output is not necessarily a smooth one, due to the specific timing of job creation and destruction. Indeed, job destruction in the home country is highest when when creation of offshored prodution lines is highest, underlining the substitution character between the two. The effects on value added are positive for the home country, as more profits are made through outsourcing, but the labour share is reduced as home employment slightly declines and a larger share of value added is generated through firm profits instead of home employment.

Other permanent shocks

[t.b.c]

5 Conclusion

Technological progress is creating a large reshuffling in global production structures. In the past, better means of transportation has fuelled the strong expansion in trade of manufactured goods. Today, advances in communication technologies allow for the great *unbundlings* as was termed by Baldwin (2006). In recent times due the global crisis and the subsequent collapse in trade and the reduction in offshoring activity, the process of offshoring has lost prominence, as noted by Blinder (2009). But in the future with a renewed pick-up in global production and very differing ajustments in labour markets and unit labour costs across countries offshoring activity will become important over the medium term. It gives firms an additional cost adjustment margin which will not remain unexploited, except if protectionist measures restrain global exchange of goods and services.

This paper addresses offshore outsourcing by modelling it as a frictional process captured by the domestic firms' need to match with foreign producers. The model manages to capture the salient dynamic patterns of global labour markets of the recent decades,

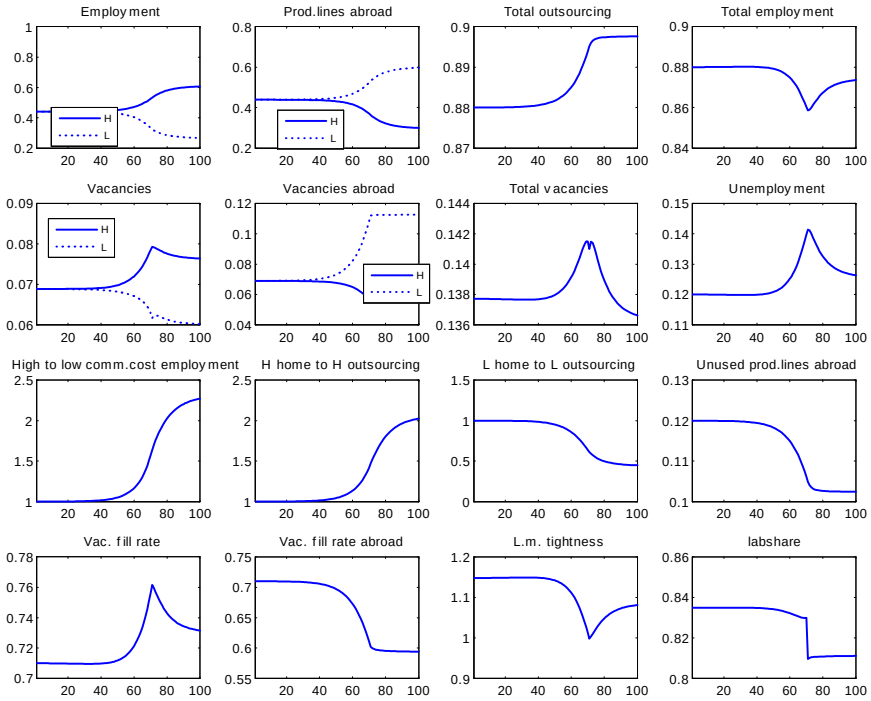

Figure 4: Permanent reduction in communication costs for low cost task. Panel A.

including trends in the adjustment of employment, wages, labour share and productivity. Our results show that the frictional model serves well as a framework to analyze global labour market developments, and which parallels the established labour market matching literature. We illustrate the interrelations between flows, creation and destruction, in domestic labour markets and and offshore outsourcing markets and the effects of the allocational interaction between the two markets on domestic employment and wages.

Our model presents the mechanisms at work in an era of global labor markets. A thorough understanding of these mechanisms is important in the current economic environment, where heavy national policy measures have been implemented and are to be implemented to enhance labour market developments in individual countries. Protectionist measures to boost domestic employment, which reduce global welfare as such, may be futile even for the domestic economy, as the intended effects may be distorted by the interrelatedness of global production chains and labour markets.

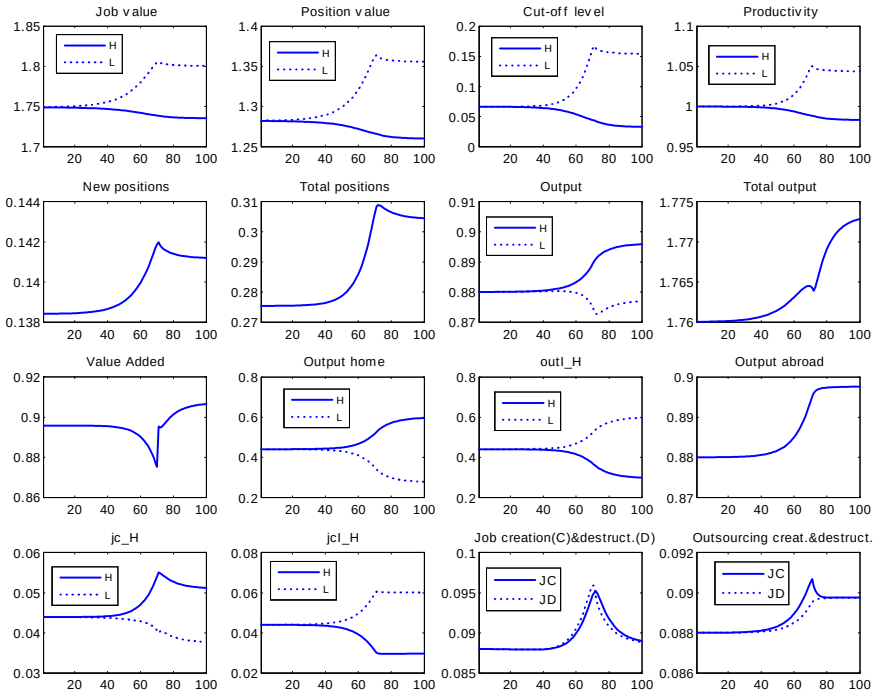

Figure 5: Permanent reduction in communication costs for low cost task. Panel B.

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6 Appendix

6.1 Model A

6.1.1 A Lagrangian

Lagrangian ($\tilde{x}_t = v_t + v_t^I$ is substituted into the last constraint)

$$\begin{aligned}\max_{\{n_t, n_{tt}^I, v_t, v_t^I, x_t, \tilde{a}_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta \Big\{ & n_t \left[A_t H_t\left(\tilde{a}_t\right) - w_t\left(a, \tilde{a}_t\right)\right] + n_t^I \left[A_t^I - \left(p_t + c_t\right)\right] \\ & - \frac{\kappa}{2} v_t^2 - \frac{\kappa^I}{2}\left(v_t^I\right)^2 - \frac{1}{2} K x_t^2 \\ & + \mu_t^J \left[\left(1-\rho^o\right)\left(1-\rho_t\right)\left(n_{t-1} + q v_t\right) - n_t\right] \\ & + \mu_t^I \left[\left(1-\rho^o\right)\left(1-\rho^{xI}\right)\left(n_{t-1}^I + q^I v_t^I\right) - n_t^I\right] \\ & + \mu_t^x \Big[\left(1-\rho^o\right)\Big[v_{t-1} + v_{t-1}^I + \rho_t n_{t-1} + \rho^{xI} n_{t-1}^I \\ & \quad - \left(1-\rho_t\right) q v_t - \left(1-\rho^{xI}\right) q^I v_t^I\Big] + x_t - v_t - v_t^I\Big]\Big\}\end{aligned} \tag{65}$$

6.1.2 A Equilibrium conditions

- Rate of exogenous and endogenous job destruction

$$\rho_t = \rho^x + \left(1-\rho^x\right) F\left(\tilde{a}_t\right) \tag{66}$$

- Total job destruction

$$\rho_t^{TOT} = \rho^o + \left(1-\rho^o\right)\rho_t \tag{67}$$

- Law of motion for employment

$$n_t = \left(1-\rho^o\right)\left(1-\rho_t\right)\left(n_{t-1} + q v_t\right) \tag{68}$$

- Law of motion for foreign employment

$$n_t^I = \left(1-\rho^o\right)\left(1-\rho^{xI}\right)\left(n_{t-1}^I + q^I v_t^I\right) \tag{69}$$

- Unemployment

$$u_t = 1 - n_t \tag{70}$$

$$[??]\ u_t = 1 - \left(1-\rho_t\right) n_t \tag{71}$$

- Foreign unused capacity

$$u_t^I = \chi_t - n_t^I \tag{72}$$

- Law of motion for positions

$$\tilde{x}_t = \left(1-\rho^o\right)\left[\tilde{x}_{t-1} + \rho_t n_{t-1} + \rho^{xI} n_{t-1}^I - \left(1-\rho_t\right) q v_t - \left(1-\rho^{xI}\right) q^I v_t^I\right] + x_t \tag{73}$$

- Job values and job destruction condition

$$\begin{aligned}
\mu_t^J &= A_t H_t(\tilde{a}_t) - w_t(a,\tilde{a}_t) + \beta \mathbb{E}_t (1-\rho^o)\left[\left(1-\rho_{t+1}\right)\mu_{t+1}^J + \rho_{t+1}\mu_{t+1}^x\right] && (74)\\
&= (1-\eta)\left[A_t H_t(\tilde{a}_t) - b\right] + \beta \mathbb{E}_t (1-\rho^o)\left[\left(1-\rho_{t+1}\right)\mu_{t+1}^J + \rho_{t+1} K x_{t+1}\right] && (75)\\
\mu_t^I &= A_t^I - (p_t + c_t) + \beta \mathbb{E}_t (1-\rho^o)\left[\left(1-\rho^{xI}\right)\mu_{t+1}^I + \rho^{xI}\mu_{t+1}^x\right] && (76)\\
\mu_t^x &= -\kappa v_t + (1-\rho^o)\left(1-\rho_{t+1}\right) q\left[\mu_t^J - \mu_t^x\right] + \beta \mathbb{E}_t (1-\rho^o)\mu_{t+1}^x && (77)\\
\mu_t^x &= -\kappa^I v_t^I + (1-\rho^o)\left(1-\rho^{xI}\right) q^I\left[\mu_t^I - \mu_t^x\right] + \beta \mathbb{E}_t (1-\rho^o)\mu_{t+1}^x && (78)\\
Kx_t &= \mu_t^x && (79)
\end{aligned}$$

$$(1-\rho^o)\frac{\partial \rho_t}{\partial \tilde{a}_t}(n_{t-1} + q v_t)\left(\mu_t^J - \mu_t^x\right) = n_t\left[A_t \frac{\partial H_t(\tilde{a}_t)}{\partial \tilde{a}_t} - \frac{\partial w_t(a,\tilde{a}_t)}{\partial \tilde{a}_t}\right] \quad (80)$$

- Equal value condition

$$(1-\rho_t) q\left[\mu_t^J - \mu_t^x\right] = \left(1-\rho^{xI}\right) q^I\left[\mu_t^I - \mu_t^x\right] \quad (81)$$

- Wage

$$w_t(\tilde{a}_t) = \eta A_t H_t(\tilde{a}_t) + (1-\eta) b \quad (82)$$

- Productivity distribution: uniform (see above, see dynare file)

$$\begin{aligned}
F(a) &= \frac{a-\underline{a}}{\bar{a}-\underline{a}}\\
f(a) &= \frac{1}{\bar{a}-\underline{a}}\\
H_t(\tilde{a}_t) &= \frac{\bar{a}+\tilde{a}}{2}\\
\frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t} &= \frac{1}{2}
\end{aligned}$$

6.1.3 A Steady state equations

Variables of interest: $\rho, n, n^I, u, u^I, v, v^I, x, \tilde{a}, \left[w\left(\tilde{a}\right), H\left(\tilde{a}\right), \mu^J, \mu^x\right]$, of which ρ and u are only supportive variables ,i.e. relevant variables are $n, n^I, v, v^I, x, \tilde{a}$

- Job filling rate

$$\bar{q} = m(\bar{u}, \bar{v})/\bar{v} = \gamma \frac{\bar{u}^{\nu}\bar{v}^{1-\nu}}{\bar{v}} = \gamma \bar{u}^{\nu}\bar{v}^{-\nu} = \gamma \left(\frac{\bar{v}}{\bar{u}}\right)^{-v} \tag{83}$$

- Rate of exogenous and endogenous job destruction

$$\bar{\rho} = \rho^o + (1-\rho^o)\,\overline{F\left(\tilde{a}\right)} \tag{84}$$

- Total job destruction

$$\bar{\rho}^{TOT} = \rho^o + (1-\rho^o)\,\bar{\rho} \tag{85}$$

- Employment

$$\bar{n} = \frac{(1-\rho^o)\,(1-\bar{\rho})\,q}{1-(1-\rho^o)\,(1-\bar{\rho})}\bar{v} \tag{86}$$

- Foreign employment

$$\bar{n}^I = \frac{(1-\rho^o)\left(1-\rho^{xI}\right)q^I}{1-(1-\rho^o)\,(1-\rho^{xI})}\bar{v}^I \tag{87}$$

- Positions

$$\bar{\tilde{x}} = \bar{v} + \bar{v}^I \tag{88}$$

- New positions

$$\bar{x}_t = [1-(1-\rho^o)]\bar{\tilde{x}} - (1-\rho^o)\left[\bar{\rho}\bar{n} + \rho^{xI}\bar{n}^I - (1-\bar{\rho})\,q\bar{v} - \left(1-\rho^{xI}\right)q^I\bar{v}^I\right] \tag{89}$$

- Unemployment

$$\bar{u} = 1-\bar{n} \tag{90}$$

$$[??]\ \bar{u} = 1-(1-\bar{\rho})\,\bar{n} \tag{91}$$

- Idle foreign production capacity

$$\bar{\chi} = 1-\bar{n}^I \tag{92}$$

- Wage

$$\bar{w}\left(\bar{\tilde{a}}\right) = \eta\bar{A}\overline{H\left(\tilde{a}\right)} + (1-\eta)\,b \tag{93}$$

- Job values

$$\bar{\mu}^J = \frac{\bar{A}\overline{H(\tilde{a})} - \bar{w}\left(\bar{\tilde{a}}\right) + \beta\left(1-\rho^o\right)\bar{\rho}K\bar{x}}{1-\beta\left(1-\rho^o\right)\left(1-\bar{\rho}\right)} \tag{94}$$

$$\bar{\mu}^I = \frac{\bar{A}^I - (\bar{p}+\bar{c}) + \beta\mathbb{E}_t\left(1-\rho^o\right)\rho^{xI}K\bar{x}}{1-\beta\left(1-\rho^o\right)\left(1-\rho^{xI}\right)} \tag{95}$$

$$\bar{\mu}^x = \frac{-\kappa\bar{v} + \left(1-\rho^o\right)\left(1-\bar{\rho}\right)q\bar{\mu}^J}{1-\left(1-\rho^o\right)\left\{\beta-\left(1-\bar{\rho}\right)q\right\}} \tag{96}$$

$$\bar{\mu}^x = \frac{-\kappa^I\bar{v}^I + \left(1-\rho^o\right)\left(1-\rho^{xI}\right)q^I\bar{\mu}^I}{1-\left(1-\rho^o\right)\left[\beta-\left(1-\rho^{xI}\right)q^I\right]} \tag{97}$$

$$\bar{\mu}^x = K\bar{x} \tag{98}$$

- Job destruction

$$\left(1-\eta\right)\left(\bar{A}\bar{\tilde{a}}-b\right) + \beta\left(1-\rho^o\right)\left(1-\bar{\rho}\right)\left(1-\eta\right)\bar{A}\left[\overline{H(\tilde{a})} - \bar{\tilde{a}}\right] = K\bar{x}\left[1-\beta\left(1-\rho^o\right)\right] \tag{99}$$

- Equal value condition for going abroad and opening vacancies at home:

- Productivity distribution: uniform

$$\begin{aligned} f(a) &= \frac{1}{\bar{a}-\underline{a}} \\ F(a) &= \frac{a-\underline{a}}{\bar{a}-\underline{a}} \\ H_t(\tilde{a}_t) &= \frac{\bar{a}+\tilde{a}}{2} \\ \frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t} &= \frac{1}{2} \end{aligned}$$

- Alternative: logistic distribution

$$\begin{aligned} f(a) &= \frac{e^{-\frac{a-\mu}{s}}}{s\left(1+e^{-\frac{a-\mu}{s}}\right)^2} \\ F(a) &= \frac{1}{1+e^{-\frac{a-\mu}{s}}} \\ H_t(\tilde{a}_t) &= s\log\left[e^{\frac{\mu}{s}} + e^{\frac{\tilde{a}_t}{s}}\right] - \frac{\tilde{a}_t}{1+e^{-\frac{\tilde{a}_t-\mu}{s}}} - 1 \\ \frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t} &= \frac{\tilde{a}_t e^{\frac{\tilde{a}_t+\mu}{s}}}{s\left(e^{\frac{\tilde{a}_t}{s}} + e^{\frac{\mu}{s}}\right)^2} \end{aligned}$$