Non-stationary Financial Risk Factors and Macroeconomic Vulnerability for the UK

Katalin Varga (MNB) & Tibor Szendrei (NIESR & HWU)

Presented by: Tibor Szendrei

6 June 2024

2024 RiskLab/BoF/ESRB Conference

Impacts of finance on Macroeconomy

- The financial system has a large impact on macroeconomic outcomes
    - Growing body of research highlights the importance of financial indicators in regression analysis of various macroeconomic variables
    - see: [Helbling et al., 2011], [Claessens et al., 2012], [Hubrich and Tetlow, 2015], [López-Salido et al., 2017], [Adrian et al., 2019], [Goulet Coulombe et al., 2022] among others
- To understand how the macroeconomy evolves, one needs to monitor and factor in stress in the financial sector
- How do we measure financial stress?

Agenda

1. Financial Stress Indices
2. Non-stationary factors
3. Variables and identified factors
4. Index Performance
5. Conclusion and way forward

Financial Stress Indices

Shrink or compress

- Financial markets are complex: many variables tracking different aspects
- The goal of an FSI is to reduce the dimensionality of the design matrix
- Shrinkage vs. compression?
    - Assumption of **sparsity** used in [Szendrei and Varga, 2023] to identify financial variables that drive euro area tail risk (see [Kohns and Szendrei, 2021] for US application)
    - Assumption of **density** used in [Chatterjee et al., 2022] to highlight the signalling potential of aggregated variables in the UK
- Financial stress is a complex phenomenon (no 2 financial crises are identical)
- We assume **dense** data and opt for compression

How to compress

Aggregation methods

- **Weighted average**
    - Simple weighted average of considered variables
- **Portfolio theory based indices**
    - Aggregate the variables using a time varying cross-correlation matrix
- **Factor models**
    - Assume the variables have a latent state that we want to capture

- Factor models are often used in macroeconometric settings while portfolio based indices are more popular in finance
- This paper will use **factor modelling** to create an FSI and compare it's performance with portfolio-theory based indices
    - First factor based stress index for the UK!

Non-stationary factors

Factors as latent risk

Dynamic Factor model

$$\begin{aligned} X_{t\times m} &= f_{t\times r}L_{r\times m} + \epsilon_{t\times m} \\ \phi(L)f_t &= d + \theta(L)u_t \end{aligned} \tag{1}$$

where r is the number of variables, m is the number of factors, f is the latent factor, and X is the design matrix.

- Compression occurs on account of $r << m$
- Factor dynamics described by VARMA(p,q) process with restrictions of [Aguilar and West, 2000] on account of factors being latent

Non-stationarity vs stationarity

- Equation (1), is a general dynamic factor model
- Just like with PCA, most common advice is to make data stationary before proceeding
- Differencing variables changes information content
  - **For example**: Term spread is often non-stationary
  - Common measure of financial stress pertaining to govt. bond market
  - What does it's (time) difference capture?
- Financial risks are described as elevated levels of variables, NOT elevated differences of variables
- Differencing variables makes it difficult to capture gradual build-up of financial stress

- A key step to make equation (1) apply to non-stationary data is to generalise the estimation of the covariance matrix
- Follow [Peña and Poncela, 2006], and take the generalised covariance matrix of X (integrated of order d):

$$C_X(k) = \frac{1}{T^{2d+D}} \sum_{t=k+1}^{T} (X_{t-k} - \bar{X})(X_t - \bar{X})' \qquad (2)$$

- $\bar{X}$ is the sample average and $D = \{0, 1\}$ depending on the existence of a drift in X_t, and k is the lag order
- Just like the empirical covariance matrix is important for stationary data, this matrix will play a key role in non-stationary factor analysis

Estimation of the model

1. **Identify the number of factors**
    - Calculate the canonical correlations for X_t and X_{t-k}
    - Apply χ^2 test to determine number of factors
2. **Calculate Generalised covariance matrix** of equation (2)
    - Estimate eigenvectors and eigenvalues
    - The initial estiamte of factor loadings is the first r eigenvectors
3. **Analyse the univariate time series properties** of the initial factor to determine order of integration
4. **Write the model in state-space form and estimate** using maximum likelihood
    - We use a Bayesian state space model for estimation.
    - See [Koop et al., 2010] for more details

Variables and identified factors

Variables and non-stationarity

Table 1: Variables for the Factor model and the P-values for the ADF test

Market	Variable	ADF test
Govt. Bond Mkt	Risk premium on 10 year bond compared to US	0.0094
	Yield on 10 year government bond - Yield on 3 month	**0.1962**
FOREX	EUR/GBP spot volatility ($\alpha = 0.94$)	**0.0578**
	USD/GBP spot volatility ($\alpha = 0.94$)	0.0388
	CHF/GBP spot volatility ($\alpha = 0.94$)	0.0059
	JPY/GBP spot volatility ($\alpha = 0.94$)	**0.0520**
	Real Effective Exchange Rate volatility ($\alpha = 0.94$)	0.0404
Capital Mkt.	CMAX of FTSE Small Cap (60 day window)	0.0010
	CMAX of FTSE 100 (60 day window)	0.0010
	CMAX of FTSE 350 (60 day window)	0.0010
	CMAX of FTSE 100 Euro (60 day window)	0.0010
	CMAX of FTSE 250 Euro (60 day window)	0.0010
	CMAX of FTSE 350 Euro (60 day window)	0.0010
Corp. Bond Mkt.	S&P UK Investment Grade Corporate Bond Index	**0.4343**
	S&P UK 3-5 Years Investment Grade Corporate Bond Index	**0.1173**
	S&P UK BBB Investment Grade Corporate Bond Index	**0.2770**
	S&P UK A Investment Grade Corporate Bond Index	**0.3169**
	S&P UK AA Investment Grade Corporate Bond Index	**0.1698**

Table 2: Factor number test: Notice similarity to cointegration test of Johansen!

	Crit. Values		S_{m-r} test given k		
r	$q_{0.05}$	$q_{0.95}$	$k = 1$	$k = 2$	$k = 3$
0	61.261	103.010	345.182*	324.672*	306.423*
1	46.595	83.675	278.449*	259.980*	243.718*
2	33.930	66.339	213.897*	198.607*	185.526*
3	23.269	50.998	149.414*	137.643*	128.074*
4	14.611	37.652	86.546*	79.304*	73.986*
5	7.962	26.296	24.593	22.257	21.464
6	3.325	16.919	2.983	2.971	2.964
7	0.711	9.488	1.985	1.975	1.971
8	0.004	3.841	0.988	0.981	0.979

Explained variation

Table 3: Explained Variance of factors

r	Expl. Variance	Cumulative
1	0.568	0.568
2	0.149	0.717
3	0.095	0.811
4	0.063	0.874
5	0.046	0.920

- The test described by [Peña and Poncela, 2006] identifies 5 factors
- Estimating 5 factors explain 92% of variation in design matrix.
- How much difference does it make to follow this procedure vs. just estimating 1 factor, or 1 factor per market (similar to [Szendrei and Varga, 2020])?

Trajectories of combined factors

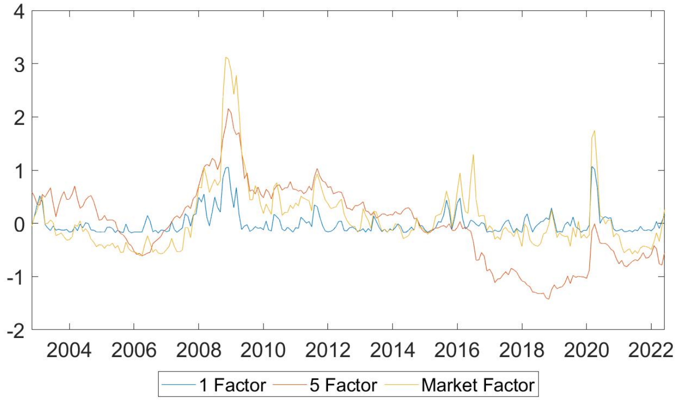

Figure 1: Trajectories of FSI's

Index Performance

Many FSI's, different stories

- As the previous figure shows: there is a potential to fit many different FSI's even if we just restrict ourselves to one methodology
- There are further FSI's for the UK which are portfolio based: SovCISS, and CLIFS
- Which FSI's are ideal for policy makers?
  - Best practice in literature is method of [Kaminsky and Reinhart, 1999], which determines the signalling potential of a variable
  - **Problem:** Method requires knowledge of crises timings
    - The purpose of an FSI is to identify stress events for high frequency data (catch-22 problem)

GaR as evaluation

- [Adrian et al., 2019] popularised the Growth-at-Risk (GaR) framework: modelling GDP growth with a value-at-risk framework
  - Downside risk of GDP can be captured by the lower quantiles of the GDP growth density
  - [Adrian et al., 2019] show that downside risk of GDP growth evolves with the state of the financial market
- The key insight, is that the proposed risk indices fit nicely into this GaR framework
  - Can use Quantile Regression of [Koenker and Bassett, 1978] to fit GaR

Construct GaR models with the different risk indices and evaluate their performance

Evaluation of FSI performance

- We fit densities for different forecast horizons
- We can use density evaluation metrics commonly used in the forecasting literature
  - [Gneiting and Ranjan, 2011] a good choice as it outlines how to utilise weighted combination of fitted quantiles to evaluate out-of-sample performance of the model:

$$qwCRPS_{(t+h)} = \int_0^1 w_i QS_{(t+h,\tau)} d\tau \tag{3}$$

  - We are interested in the out-of-sample performance of fitting the left tail: place more weight on lower quantiles' forecast performance
- Since FSI's are fast moving, we construct Monthly GDP estimates for the UK using the method of [Koop et al., 2023]

Results (Individual factors): Short forecast horizon

Table 4: Performance of different FSI's

	1 Factor	5 Factors	Market Factors	CLIFS	SovCISS
h=1					
AIC	76.001	**75.227**	75.616	76.857	76.647
BIC	76.280	**76.064**	76.313	77.136	76.926
w_{centre}	**0.054**	0.056	0.056	0.055	0.055
w_{left}	**0.085**	0.087	0.088	0.089	0.088
h=3					
AIC	92.307	**90.488**	91.242	93.481	92.841
BIC	92.587	**91.329**	91.943	93.762	93.121
w_{centre}	0.132	**0.131**	0.134	0.136	0.132
w_{left}	**0.210**	**0.210**	0.218	0.230	0.222

Results (Individual factors): Long forecast horizon

Table 5: Performance of different FSI's

	1 Factor	5 Factors	Market Factors	CLIFS	SovCISS
h=6					
AIC	99.394	**96.114**	97.299	99.854	98.253
BIC	99.677	**96.963**	98.007	100.137	98.536
w_{centre}	0.189	**0.178**	0.189	0.198	0.186
w_{left}	0.324	**0.297**	0.324	0.340	0.310
h=12					
AIC	101.900	98.965	99.897	101.254	**97.207**
BIC	102.188	99.830	100.618	101.542	**97.496**
w_{centre}	0.237	0.221	0.233	0.231	**0.184**
w_{left}	0.399	0.381	0.397	0.385	**0.306**

Results (Combined factors): Short forecast horizon

Table 6: Performance of combined factors (Note: factor combination methodology has impact on results)

	1 Factor	5 Factors (Combined)	Market Factors (Combined)	CLIFS	SovCISS
h=1					
AIC	**76.001**	76.827	76.214	76.857	76.647
BIC	**76.280**	77.105	76.493	77.136	76.926
w_{centre}	**0.054**	0.055	0.055	0.055	0.055
w_{left}	**0.085**	0.089	0.086	0.089	0.088
h=3					
AIC	**92.307**	92.839	92.343	93.481	92.841
BIC	**92.587**	93.120	92.624	93.762	93.121
w_{centre}	0.132	**0.131**	0.133	0.136	0.132
w_{left}	**0.210**	0.223	0.218	0.230	0.222

Results (Combined factors): Long forecast horizon

Table 7: Performance of combined factors (Note: factor combination methodology has impact on results)

	1 Factor	5 Factors (Combined)	Market Factors (Combined)	CLIFS	SovCISS
h=6					
AIC	99.394	98.658	98.669	99.854	**98.253**
BIC	99.677	98.942	98.952	100.137	**98.536**
w_{centre}	0.189	**0.186**	0.188	0.198	**0.186**
w_{left}	0.324	0.324	0.324	0.340	**0.310**
h=12					
AIC	101.900	100.142	101.774	101.254	**97.207**
BIC	102.188	100.430	102.062	101.542	**97.496**
w_{centre}	0.237	0.213	0.239	0.231	**0.184**
w_{left}	0.399	0.380	0.401	0.385	**0.306**

Conclusion and way forward

Concluding thoughts and way forward

- Financial stress is captured by the level of financial variables and not the difference of them: This is best captured by non-stationary risk factors
- Factor based financial risk indices are potent
    - Captures downside risk in the short run
    - SovCISS portrays very good performance for longer term horizons.
- Methodologies are complimentary and not substitutes
    - Ideal to track the two FSI's together
- **Way forward:** Introducing information from network and spatial domain to allow to track spillovers

Thank you for your attention!

Paper available at: `https://arxiv.org/abs/2404.01451`

Adrian, T., Boyarchenko, N., and Giannone, D. (2019).
Vulnerable growth.
American Economic Review, 109(4):1263–1289.

Aguilar, O. and West, M. (2000).
Bayesian dynamic factor models and portfolio allocation.
Journal of Business & Economic Statistics, 18(3):338–357.

Chatterjee, S., Chiu, C.-W., Duprey, T., and Hacıoğlu-Hoke, S. (2022).
Systemic financial stress and macroeconomic amplifications in the united kingdom.
Oxford Bulletin of Economics and Statistics, 84(2):380–400.

References II

- Claessens, S., Kose, M. A., and Terrones, M. E. (2012).
  How do business and financial cycles interact?
  Journal of International Economics, 87(1):178–190.
- Gneiting, T. and Ranjan, R. (2011).
  Comparing density forecasts using threshold-and quantile-weighted scoring rules.
  Journal of Business & Economic Statistics, 29(3):411–422.
- Goulet Coulombe, P., Leroux, M., Stevanovic, D., and Surprenant, S. (2022).
  How is machine learning useful for macroeconomic forecasting?
  Journal of Applied Econometrics, 37(5):920–964.

Helbling, T., Huidrom, R., Kose, M. A., and Otrok, C. (2011).
Do credit shocks matter? A global perspective.
European Economic Review, 55(3):340–353.

Hubrich, K. and Tetlow, R. J. (2015).
Financial stress and economic dynamics: The transmission of crises.
Journal of Monetary Economics, 70:100–115.

Kaminsky, G. L. and Reinhart, C. M. (1999).
The twin crises: the causes of banking and balance-of-payments problems.
American economic review, 89(3):473–500.

Koenker, R. and Bassett, G. (1978).
Regression quantiles.
Econometrica: journal of the Econometric Society, pages 33–50.

References IV

Kohns, D. and Szendrei, T. (2021).
Decoupling shrinkage and selection for the Bayesian quantile regression.
arXiv preprint arXiv:2107.08498.

Koop, G., Korobilis, D., et al. (2010).
Bayesian multivariate time series methods for empirical macroeconomics.
Foundations and Trends® in Econometrics, 3(4):267–358.

Koop, G., McIntyre, S., Mitchell, J., and Poon, A. (2023).
Reconciled estimates of monthly gdp in the united states.
Journal of Business & Economic Statistics, 41(2):563–577.

López-Salido, D., Stein, J. C., and Zakrajšek, E. (2017).
Credit-market sentiment and the business cycle.
The Quarterly Journal of Economics, 132(3):1373–1426.

Peña, D. and Poncela, P. (2006).
Nonstationary dynamic factor analysis.
Journal of Statistical Planning and Inference, 136(4):1237–1257.

Schorfheide, F. and Song, D. (2015).
Real-time forecasting with a mixed-frequency var.
Journal of Business & Economic Statistics, 33(3):366–380.

Szendrei, T. and Varga, K. (2020).
Fiss–a factor-based index of systemic stress in the financial system.
Russian Journal of Money and Finance, 79(1):3–34.

Szendrei, T. and Varga, K. (2023).
Revisiting vulnerable growth in the euro area: Identifying the role of financial conditions in the distribution.
Economics Letters, page 110990.

Appendix

QR to GaR

- The idea of GaR is to fit the following model:

$$y_{t+h} = \beta_0(\tau) + \beta_1(\tau)y_t + \beta_2(\tau)FSI_t + \varepsilon_{t+h} \tag{4}$$

- here τ is the quantile of interest
- So we can fit GaR for the different FSI's at various τ values below the median
- The in- and out-of-sample performance can tell us about how well the FSI performs at uncovering financial stress that is relevant for macro-financial linkages
  - Macroprudential policy makers goal is to minimise the impact of financial turbulence on macroeconomy
  - Unlike threshold methods, quantile regression does not need the number of regimes to be specified

Quantile Regression

- GaR is conceptually simple, but:
- How do we estimate the $\beta(\tau)$?

Quantile Regressions "Pinball" Loss Function

Quantile regression aims to fit the following model: $y_t = x_t'\beta(\tau) + \varepsilon_t$.
Here $\tau \in (0,1)$ is the quantile of interest.
[Koenker and Bassett, 1978] show that one can obtain estimates of $\beta(\tau)$ by optimising the following function:

$$\hat{\beta}(\tau) = \underset{\beta(\tau)}{argmax} \sum_{t=1}^{T-h} \Big[I(y_{t+h} \geq x_t'\beta(\tau))|y_{t+h} - x_t'\beta(\tau)|\tau \\ + I(y_{t+h} < x_t'\beta(\tau))|y_{t+h} - x_t'\beta(\tau)|(1-\tau) \Big] \quad (5)$$

Back to Main Text

Monthly GDP

- Start from quarterly GDP values based on expenditure and production approach
  - Consider these as noisy measures of true GDP

$$
\begin{bmatrix} GDP_{P,t} \\ GDP_{E,t} \end{bmatrix} = 1_{2\times 1} GDP_t + \begin{bmatrix} \varepsilon_{P,t} \\ \varepsilon_{E,t} \end{bmatrix} \qquad (6)
$$

$$
GDP_t = \rho GDP_{t-1} + \varepsilon_{G,t}
$$

- [Koop et al., 2023] use **noise restriction**: assumption that the variance of true GDP is less than the variance some of its noisy observation

$$
\xi_i = \frac{var(GDP)}{var(GDP_i)} \qquad (7)
$$

- We assume that $0.35 < \xi_P$, $\xi_E < 1.15$ based on the empirical variance of production- and expenditure-based GDP

Monthly Frequency

- The mixed frequency model can be summarised as:

$$\begin{aligned} y_t &= (X_t', U_t, GDP_t, GDP_{P,t}, GDP_{E,t}) \\ y_t^Q &= \Delta_3 \ln(Y_t) \end{aligned} \tag{8}$$

- Y_t^Q is the quarterly variable observed every third month
- U_t is the unemployment rate
  - depends on GDP but not on GDP_P or GDP_E
- X_t' is a set of monthly explanatory variables
  - variables of [Schorfheide and Song, 2015] and [Koop et al., 2023]
  - retail sales, inflation, industrial production, base rate, short-term interest rates, long-term interest rates, and stock prices
- We use GDP_P, since it does not contain taxes

Final GDP measures

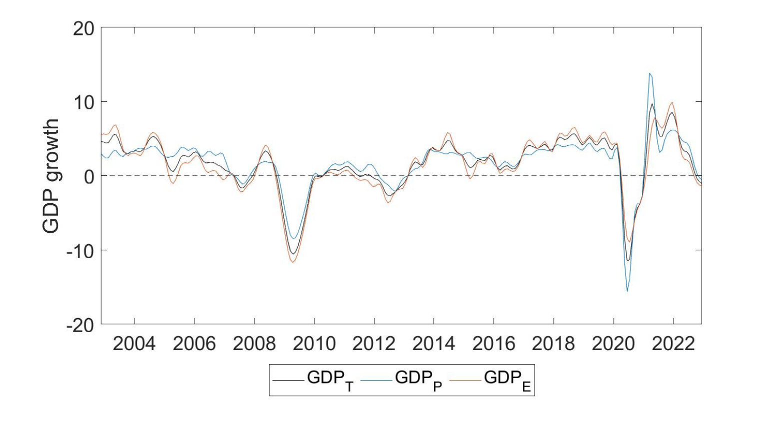

Figure 2: The three measures of Monthly GDP

Back to Main Text