The not-so-hidden risks of bank runs and fire-sales with ‘hidden-to-maturity’ accounting

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Abstract

We build a model that captures bank run and fire-sale risk, i.e., a reduced potential to raise capital from liquidity buffers under stress. The model setup is inspired by the Silicon Valley Bank (SVB) meltdown in March 2023. Consequently, we use the model to explain how vulnerabilities in SVB's balance sheet evolved and we show which characteristics of the balance sheet are important to monitor in order for banking system regulators to assess the run risk. By bringing a time series of SVB's balance sheet data to our model, we can demonstrate how the change in the funding and respective asset composition—increasingly relying on held-to-maturity accounting standards masking revaluation losses in securities portfolios—made SVB prone to run risk. The parsimonious framework that we propose can serve as a supervisory analysis tool to monitor build-up of vulnerabilities in banks' balance sheets.

1 Introduction

As documented in Granja (2023), the years 2021–22 saw a substantial rise in banks' reliance on held-to-maturity (HtM) securities, effectively allowing them to ‘hide’ unrealized losses, e.g., in the face of raising rates negatively impacting bond prices as with Silicon Valley Bank (SVB). For this reason, the HtM framework has become described as a form of ‘hidden-to-maturity’ accounting.

Worryingly, Granja (2023) also shows how the HtM accounting rules were more frequently applied by less capitalised banks with significant (run-prone) uninsured deposits in order to immunize their capital from revaluation of securities held on-balance. In times of distress or when market expectations change, this may be detrimental to those banks' health and—as the March 2023 banking turmoil showed—to financial stability as a whole.

In this paper, we build a stylized model to explain bank runs by banks' financial conditions, as observed by depositors, and to measure vulnerabilities of banks stemming from their balance sheet composition, in particular as it pertains to the classification of HtM assets. We use the model to compute deposit withdrawals in equilibrium that considers the share of run-prone uninsured deposits in the bank's funding, a pool of liquid resources, and those assets that can be mobilized with an impact on bank's profit & loss accounts, firstly because of liquidation frictions related to a price-mediated channel of contagion, when banks need to sell securities to raise cash, and, secondly, due to accounting rules requiring banks to fully mark-to-market HtM portfolios when any such securities are made available for sale.

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In this way, our model provides a parsimonious theoretical framework that can help regulators understand some of the key drivers behind run related instabilities, in particular identifying threshold ratios of banks' balance sheets that delineate stable financial conditions from those conducive to bank runs. The model also explains why changing financial ratios, such as the share of uninsured deposits or HtM securities in total assets, may result in abrupt jumps in banks' solvency or liquidity conditions. Throughout this work, we demonstrate various insights from our framework by running numerical case studies based on balance sheet information from SVB in the run up to its failure in March 2023.

Kim et al. (2023) provide evidence that banks' use of HtM is mainly guided by attempts to optimise around capital requirements and accounting measures such as net income and owner's equity (especially when there are concerns about the equity becoming negative). In general, this suggests that banks will maximise the amount of HtM they can reasonably hold subject to the loss of flexibility that this entails in terms of selling and re-classification of assets. Crucially, the U.S. Generally Accepted Accounting Principles require that a bank has both the positive intent and the *ability* to hold securities until maturity if they are to be classified as HtM. It is only if this ability is taken seriously that there can be any case for HtM accounting to have a stabilising effect, as the disregard of (sudden) deviations from the book value may then be justified by the bank's ability to not resort to remarking. While the findings of Kim et al. (2023) show that the intent and/or ability often seem to be lacking, one would expect the banks to at least assign some weight to avoid being forced into re-marking their HtM portfolio in most scenarios. Firstly, this is what a bank should do if (at least partially) complying with the HtM rules and, secondly, internal risk management should aim to avoid having to recognize large unrealized losses and signalling bad conditions by re-marking as well as loosing the ability to reclassify the given assets as HtM again. To capture this, we formulate a simple optimisation problem: for a given size of the balance sheet and a random initial market price, the bank maximizes the amount of assets classified as HtM subject to a chance constraint on the need to re-mark these HtM assets.

This optimisation problem presents a distilled view of the main decision variables and can provide a simple explanation for why banks hold given levels of HtM assets. The way our model is set up makes it a highly tractable problem that is easy to calibrate and implement numerically. Given observable balance sheet data, the optimisation problem can therefore be used to imply a particular risk tolerance, as measured by the threshold probability of the chance constraint, which can help shed light on a bank's ability to hold the assets to maturity.

We bring the model to balance sheet data of the SVB, based on publicly available sources, to assess build-up of vulnerabilities of SVB ahead of its default in March 2023. The key results are as follows. Financial standing of the SVB was very sensitive to fire sale conditions, i.e., the sensitivity of prices of securities to liquidated volumes of securities. Unrealised losses in SVB's balance sheet, related to HtM accounting, hid important weaknesses of the banks and the model can indicate periods prior to the March 2023 meltdown when the situation of the bank became dire. It is clear that fostering banks' insured funding sources would be an important policy, or supervisory, tool to prolong the runway toward default but would not avert the default completely. In general, the algorithm 3.2 which we derive from our model to characterise default conditions offers an easy monitoring tool to assess vulnerabilities.

The remainder of the paper is structured as follows: in the next subsection, we briefly describe the most relevant literature. Then, we introduce the model. In the subsequent section, we discuss case studies of the SVB in light of the introduced model. Finally, we talk about how to better understand allocation of assets to AfS and HtM accounting portfolios.

1.1 Related literature

Bank runs associated with risky projects and short-term, flighty funding have been studied extensively since the first comprehensive model of Diamond and Dybvig (1983) explaining banks' fragility. They tackle the duality of multiple equilibria in banks' funding conditions that may arise, implying run or no-run on banks. The mechanism described by them is about interplay of short-term (impatient) creditors, risky return on securities held by banks (the return on the projects can only happen in the future and if banks need to liquidate securities to meet depositors, the bank would be short of proceeds from those projects and would run out of money). Heterogeneous beliefs would create partial runs, with a fraction of depositors withdrawing cash. The original model speaks about very traditional banking business model: client (household deposits) and long-term investments (mortgage or investment loans to finance construction or tangible assets). In fact, the run risk holds for a variety of business models of intermediaries. For instance, Uhlig (2010) demonstrates runs conducted by banks on other banks holding short term and tradable assets, like asset-backed securities, a phenomenon observed during the Global Financial Crisis.

The assumptions on the beliefs of depositors and the banks are key to understand how the equilibria may arise. Global games (Morris and Shin, 2003) was a breakthrough technique to capture the idea that depositors may be pushed to withdraw funding because of their belief that others are taking such actions. This rationalizes, or endogenizes, the funding shocks.

Studies on bank-run mechanism depict certain important drivers that we also cover in our model. First, fire sale happens when an institution sells under pressure and below the fundamental value of the assets, e.g., because of liquidity or solvency constraints. Their role in runs were studied by Diamond and Rajan (2011). Fire sale may depress asset prices so much that a bank holding them or selling them becomes insolvent. This could precipitate a run on the bank, causing additional pressure for banks to liquidate assets and further depressing their prices. Second, leverage is an important indicator of potential vulnerabilities that depositors may take into account when assessing safety of their deposits. We put leverage at the center of our model as a key object determining bank runs in equilibrium. Third, accounting rules, e.g., fair value vs amortised cost accounting, and regulatory regimes, e.g., provisioning for credit risk, can induce certain behaviours of market participants that contribute to a build-up of financial vulnerabilities. We capture those as well to describe balance sheet elements that can be easily mobilized to raise liquidity but are market to market instantaneously and those that can only be utilized as a last-resort source of liquidity but are more immune to market volatility.

Despite the topic being researched extensively and a stringent liquidity regulation and policy intervention frameworks in place, liquidity risk forcefully materializes again and again. In March 2023, Silicon Valley Bank became a textbook example of a bank run. When SVB imploded, authorities looked deeper into the unrealised losses that were the root cause of its meltdown (see, e.g., relevant FDIC and ECB reports). As shown by Drechsler et al. (2023), runs may occur in the rising interest rate environment since hedging may not be able to fully and at the same time eliminate interest and liquidity risk given the negative convexity of bank deposit franchise value and typical long duration of bank assets. The authors focus on a question about hedging of either liquidity or interest rate risk to prevent runs, which is different to our primary goal to characterize liquidity and solvency vulnerabilities in the balance sheet. Dependence of the fragility of banks on changes of rates in different rate environments (low vs high) was studied by Ahnert et al. (2023) demonstrating the risk of runs increases more when rates rise from low levels. SVB was exposed to this risk of rising interest rates, and from low levels observed during the COVID-19 pandemic, but, additionally those losses were hidden given the accounting treatment of held-to-maturity assets. One lesson learned from March 2023 is that the accounting rules masking the adverse changes in fair value of banks assets and the very unstable funding sources (concentrated and easy to call back) may create conditions for the outflow of deposits to happen. Subsequent fire sales may exacerbate banks' solvency and the overall market conditions. As shown by Liu (2023), small shocks to the balance sheet of banks may be amplified to systemic events.

We contribute to the growing literature on the impact of accounting standards on financial stability. Reporting frameworks, but also incentives, play a role in market participants' assessment of financial conditions (Bischof et al., 2021). Especially relevant are the recent works Granja (2023); Kim et al. (2023) on HtM accounting and how it is employed by banks in times of stress, as discussed above. We study policy options to avert the bank run risk, esp., looking at deposit insurance and differently, e.g., to Altermatt et al. (2022) who study the role of redemption penalties. In particular, we build a model that gives supervisors a tool to easily watch for banks approaching those cliffs and allow for supervisory actions to preemptively mitigate related risks.

2 Balance Sheet Construction and Model Setup

We shall consider the simplest possible formulation of a balance sheet that allows us to capture the features we are interested in. First of all, this entails two classes of liabilities (i.e., deposits): they can be either insured $L_I \geq 0$ or uninsured $L_U \geq 0$ with the total liabilities given by $L := L_I + L_U$. Next, we shall assume that assets of the bank can be one of three types: liquid, illiquid but marketable, or illiquid and nonmarketable. Finally, in the case of illiquid but marketable assets, these may be classified as either available-for-sale or held-to-maturity. Beyond this classification, these illiquid but marketable securities are subject to the same market price. In summary, the stylized balance sheet will consist of the following four asset classes:

(i) liquid (cash) assets $x \geq 0$;

(ii) available-for-sale (AfS) illiquid assets $s \geq 0$ with an initial mark-to-market value of sp for some unit price $p > 0$;

(iii) held-to-maturity (HtM) illiquid assets $h \geq 0$ valued in full (despite being subject to the same market price as the AfS assets); and

(iv) nonmarketable illiquid assets $\ell \geq 0$.

With the above notation, the total assets are given by $A = x + sp + h + \ell$. The bank's equity is then then difference between this value and the total liabilities (L). These quantities determine the initial balance sheet before any consideration of a run.

Remark 2.1. For now, we take the classification of AfS versus HtM as fixed and given. In Section 5, we address this allocation by way of a simple optimisation problem, as discussed in the introduction.

Following Banerjee and Feinstein (2021), we assume that the illiquid, but marketable, holdings are subject to price impacts if they need to be sold. The mark-to-market value of these assets is given by the inverse demand function $f : [0, s+h] \to [0, p]$ for the initial price $p > 0$. As these liquidations are realized, the bank realizes the volume weighted average price (VWAP) $\bar{f}(\gamma) := \frac{1}{\gamma}\int_0^\gamma f(t)dt$, for $\gamma \in (0, s+h]$, with $\bar{f}(0) := p$. Note that $\bar{f}$ is continuous at $\gamma = 0$ if f is continuous there. In this way, any unsold AfS assets are valued at the price determined by f, any sold AfS assets are valued at the price determined by $\bar{f}$, and any HtM assets are (initially) valued at a fixed price of 1. Similarly, we will assume throughout that the liquid (x) and nonmarketable (ℓ) assets have fixed value throughout this study.

The composition of the initial balance sheet and an example of a realized balance sheet after withdrawals are illustrated in Figure 1. Based on the balance sheet, the *uninsured* depositors will withdraw

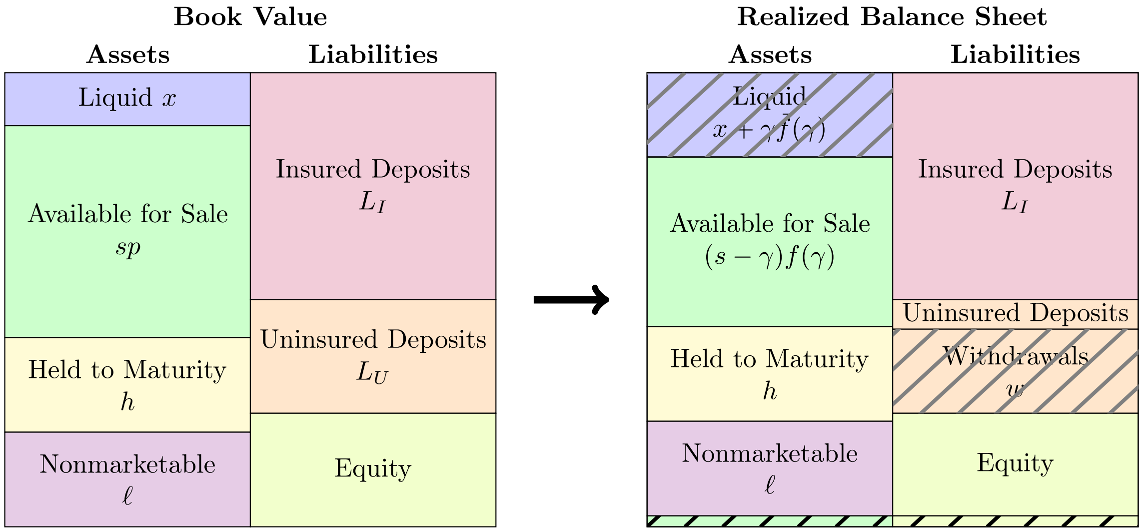

Figure 1: Stylized book and balance sheet for the bank subject to withdrawal risks where held to maturity assets need not be sold.

their funds following a leverage targeting strategy. The total withdrawals that result from this are denoted by $w \in [0, L_U]$. Specifically, the uninsured investors have a maximum leverage ratio $\lambda_{\max} > 1$ that they are willing to accept before withdrawals are initiated. The actual leverage ratio λ is defined as the ratio of assets over equity, accounting for withdrawals and the corresponding losses that must be recognized on the balance sheet. Notably, leverage is one of the key financial indicators determining banks' stability and has been used as an important variable in seminal bank run models (Gertler and Kiyotaki, 2015).[1] Furthermore, the leverage ratio is one of two key solvency indicators that is regulated by capital standards, most importantly by Basel III regulation introduced after the Global Financial Crisis. In particular, banks should keep it about a regulatory minimum and typically retain a voluntary buffer, so as to minimize the risk that the leverage ratio falls below requirements.[2] For these reasons, we focus on the leverage ratio as the sole signal tracked by depositors. Naturally, the *insured* depositors leave their funds at the bank even in a stress scenario—in particular, a bank run—due to the guarantee of recovery in case of a bank failure.

Assumption 2.2. *The inverse demand function $f : [0, s+h] \to (0, p]$ is non-increasing with initial price $f(0) = p$, where $p \in (0, 1]$.*

Assumption 2.3. *We assume $L_U > 0$ as no withdrawals would occur otherwise.*

For modelling purposes, we stress that the quantities L_I, L_U, and L remain fixed, as they capture the liabilities of the initial balance sheet, before a run. The uninsured liabilities after withdrawals are then given by $L_U - w$. Since nothing is withdrawn from insured liabilities L_I, the total liabilities thus become $L - w$. Writing $A(w, \gamma)$ for the total assets (with recognized losses) as a function of the withdrawals w and the quantity of marketable securities sold γ, we can express the leverage ratio $\lambda = \lambda(w, \gamma)$ as

$$\lambda = \frac{\text{Assets}}{\text{Equity}} = \frac{A(w,\gamma)}{A(w,\gamma) - (L-w)}, \tag{1}$$

where

$$A(w,\gamma) = x + \gamma \bar{f}(\gamma) + [s-\gamma]^+ f(\gamma) + [h - (\gamma - s)^+](\mathbb{I}_{\{\gamma \le s\}} + f(\gamma)\mathbb{I}_{\{\gamma > s\}}) + \ell - w,$$

[1]It is one of several key financial stress indicators (Duca and Peltonen, 2013).
[2]See https://www.bis.org/basel_framework/standard/LEV.htm

for the given values of x, s, h, and ℓ.

In order to satisfy the withdrawals, the bank needs to raise liquid assets by (potentially) selling its illiquid, but marketable, holdings. Specifically, the bank sells $\gamma \in [0, s+h]$ so that $x + \gamma \bar{f}(\gamma) \geq w$ (if possible).[3] Based on the assumed balance sheet, if $\gamma \leq s$ then we assume that all liquidated assets were AfS assets. However, if $\gamma > s$ then the bank is liquidating HtM assets as well. Formally, if any HtM assets are to be liquidated then that entire block of assets is immediately redenominated as AfS and marked accordingly. Summarizing these two cases:

(i) if $\gamma \leq s$ then the bank holds $x + \gamma \bar{f}(\gamma)$ in liquid assets, $(s - \gamma) f(\gamma)$ in AfS assets, and h in HtM assets;

(ii) if $\gamma > s$ then the bank holds $x + \gamma \bar{f}(\gamma)$ in liquid assets, $(s + h - \gamma) f(\gamma)$ in AfS assets, and 0 HtM assets.

However, in satisfying the withdrawals, the bank may fail due to having insufficient liquidity or insufficient equity. We call these cases illiquidity and insolvency respectively.

(i) **Illiquidity**: The bank cannot meet withdrawals: $w \geq x + [s+h]\bar{f}(s+h)$ or, equivalently, $\gamma = s+h$.

(ii) **Insolvency**: The bank has negative equity: $x + \gamma \bar{f}(\gamma) + [s - \gamma]^+ f(\gamma) + [h - (\gamma - s)^+](\mathbb{I}_{\{\gamma \leq s\}} + f(\gamma)\mathbb{I}_{\{\gamma > s\}}) + \ell \leq L$.

Remark 2.4. If there are no price impacts on the illiquid asset, i.e., $f \equiv p$, then insolvency can only occur at the moment that the HtM assets are re-marked as AfS assets.

For our analysis, we shall need a final assumption on the behavior of the realized balance sheet. Recall $\lambda_{\max} > 1$. As a function of the quantity sold, we require that the rate increase in the realized value of the (total) assets sold is always larger, by a factor of $1 - \frac{1}{\lambda_{\max}} > 0$, than the corresponding rate of decrease in the market value of the remaining unsold assets. More precisely, we impose the following technical assumption on the inverse demand function f.

Assumption 2.5. *For the remainder of this work, we will assume that the mapping* $\gamma \in [0, s+h] \mapsto \gamma \bar{f}(\gamma) + (1 - \frac{1}{\lambda_{\max}})(s + h - \gamma) f(\gamma)$ *is strictly increasing.*

Lemma 2.6. *Suppose the inverse demand function f is differentiable. Then Assumption 2.5 holds if and only if the differential inequality*

$$f(\gamma) > (1 - \lambda_{\max})(s + h - \gamma) f'(\gamma)$$

is satisfied, for all $\gamma \in [0, s+h]$.

Proof. This follows from the definition of $\bar{f}$ and by differentiating the given function, insisting on its derivative being strictly positive. □

Remark 2.7. Under Assumption 2.5, we get that $\gamma \mapsto \gamma \bar{f}(\gamma) + (1 - \frac{1}{\lambda_{\max}})(\bar{s} - \gamma) f(\gamma)$ is strictly increasing on $[0, \bar{s}]$, for any $\bar{s} \in (0, s+h]$. In particular, this holds for $\bar{s} \in \{s, s+h\}$ which we shall make use of in Section 3. At the same time, we stress that the map $\gamma \mapsto \gamma \bar{f}(\gamma) + (\bar{s} - \gamma) f(\gamma)$ is instead non-increasing on $[0, \bar{s}]$, for any $\bar{s} \in (0, s+h]$, as one can readily verify (e.g. arguing as in Lemma 2.6).

We conclude this section by highlighting two common examples of inverse demand functions and outlining the parameter choices for which our assumptions are satisfied.

[3] We impose a no short selling constraint so that $\gamma \leq s + h$ throughout.

Example 2.8. Take $f(\gamma) := p(1-b\gamma)$ as in, e.g., Greenwood et al. (2015). Then $\bar{f}(\gamma) = p(1-\frac{b}{2}\gamma)$. Naturally, $b \leq 1/(s+h)$ and hence Assumptions 2.2 is satisfied. By Lemma 2.6, Assumption 2.5 holds if and only if either $b < 1/(s+h)$ for $\lambda_{\max} \in (1,2)$ or $b < 1/[(\lambda_{\max}-1)(s+h)]$ for $\lambda_{\max} \geq 2$.

Example 2.9. Take $f(\gamma) = p\exp(-b\gamma)$ as in, e.g., Cifuentes et al. (2005). Then $\bar{f}(\gamma) = \frac{p(1-\exp(-b\gamma))}{b\gamma}$ for $\gamma > 0$ and $\bar{f}(0) = p$. Naturally, $b \geq 0$, so Assumption 2.2 holds. By Lemma 2.6, Assumption 2.5 holds if and only if $b < 1/[(\lambda_{\max}-1)(s+h)]$.

3 Clearing Prices and Bank Failures

Consider the balance sheet structure introduced in Section 2. We can now formulate the resolution of a (potential) bank-run and the accompanying fire-sales as the solution to a simple clearing problem for the amount of withdrawals w and the quantity of marketable securities sold γ.

We formalise this as the search for fixed points of the mapping $\Phi : [0, L_U]\times[0, s+h] \to [0, L_U]\times[0, s+h]$ defined by $\Phi = (\Phi_w, \Phi_\gamma)$, where

$$\begin{aligned} \Phi_w(\gamma^*) &= L_U \wedge \Big[\lambda_{\max} L - (\lambda_{\max}-1)(x + \gamma^*\bar{f}(\gamma^*) + [s-\gamma^*]^+ f(\gamma^*) \\ &\qquad + [h-(\gamma-s)^+](\mathbb{I}_{\{\gamma^*\leq s\}} + f(\gamma^*)\mathbb{I}_{\{\gamma^*>s\}}) + \ell)\Big]^+ \end{aligned} \tag{2}$$

$$\Phi_\gamma(w^*, \gamma^*) = [s+h] \wedge \frac{(w^*-x)^+}{\bar{f}(\gamma^*)}. \tag{3}$$

Here, equation (2) is enforcing the leverage targeting strategy for the withdrawals of the depositors, while (3) gives the quantity sold necessitated by the corresponding withdrawal requests. A pair $(w^*, \gamma^*) \in [0, L_U] \times [0, s+h]$ is then a clearing solution if and only if it is a fixed point of Φ, meaning that we have

$$(w^*, \gamma^*) = \Phi(w^*, \gamma^*) = (\Phi_w(\gamma^*)\ ,\ \Phi_\gamma(w^*, \gamma^*)), \tag{4}$$

provided the bank is solvent, i.e.

$$x + \gamma^*\bar{f}(\gamma^*) + [s-\gamma^*]^+ f(\gamma^*) + [h-(\gamma^*-s)^+](\mathbb{I}_{\{\gamma^*\leq s\}} + f(\gamma^*)\mathbb{I}_{\{\gamma^*>s\}}) + \ell > L. \tag{5}$$

If (w^*, γ^*) satisfies (4), but violates (5), then the bank is insolvent. In that case, the values (w^*, γ^*) correspond to the run having occurred and the bank only subsequently being declared insolvent. This is arguably more in line with the timeline of events in an actual run, but one can of course also look for the amount of liquidations γ that first induces technical insolvency by violating (5) during the run.

For clearing solutions corresponding to a run (i.e., $w^* > x$) without causing illiquidity (i.e., $\gamma^* < s+h$), we have $w^* = x + \gamma^*\bar{f}(\gamma^*)$ with all withdrawals being met, solvency issues aside. On the other hand, illiquidity corresponds to clearing solutions with a quantity sold $\gamma^* = s+h$ and withdrawals $w^* \geq x + (s+h)\bar{f}(s+h)$. When a bank is left illiquid, the value of w^* reflects the withdrawal requests and not the actualized withdrawals, as the bank would generally not be able to cover all requests.

Proposition 3.1 (Existence)**.** *There exist minimal and maximal clearing solutions* $(w^\downarrow, \gamma^\downarrow) \leq (w^\uparrow, \gamma^\uparrow)$.

Throughout, we work with the minimal solution, as this is the best case for the bank and represents the solution that a run would most naturally arrive at. We have the following (exhaustive) algorithm for finding the minimal clearing solution.

Proposition 3.2 (Clearing algorithm)**.** *The minimal clearing solution* $(w^\downarrow, \gamma^\downarrow)$ *is determined by the following algorithm:*

1. **(No sales)** *If either* $L_U \leq x$ *or* $\lambda_{\max} L - (\lambda_{\max} - 1)(x + sp + h + \ell) \leq x$*, then* $\gamma^{\downarrow} = 0$ *and* $w^{\downarrow} = L_U \wedge [\lambda_{\max} L - (\lambda_{\max} - 1)(x + sp + h + \ell)]^+$*. Else, continue to next step.*

2. **(Run without re-marking HtM I)** *If*

$$
\begin{aligned}
&L - x - (1 - \tfrac{1}{\lambda_{\max}})(h + \ell) \in [(1 - \tfrac{1}{\lambda_{\max}})sp, s\bar{f}(s)], \quad \textit{and} \\
&L_U \geq \lambda_{\max} L - (\lambda_{\max} - 1)(x + \gamma^* \bar{f}(\gamma^*) + (s - \gamma^*)f(\gamma^*) + h + \ell), \quad \textit{for} \\
&\gamma^* \bar{f}(\gamma^*) + (1 - \tfrac{1}{\lambda_{\max}})(s - \gamma^*)f(\gamma^*) = L - x - (1 - \tfrac{1}{\lambda_{\max}})(h + \ell), \quad \gamma^* \in [0, s],
\end{aligned}
$$

then $\gamma^{\downarrow} = \gamma^*$ *and* $w^{\downarrow} = x + \gamma^* \bar{f}(\gamma^*) \in (x, L_U)$*. Else, continue to next step.*

3. **(Run without re-marking HtM II)** *If* $L_U \in (x, x + s\bar{f}(s)]$ *and* $L_I \geq (1 - \frac{1}{\lambda_{\max}})[(s - \gamma^*)f(\gamma^*) + h + \ell]$ *for* $\gamma^* \in [0, s]$ *solving* $\gamma^* \bar{f}(\gamma^*) = L_U - x$*, then* $\gamma^{\downarrow} = \gamma^*$ *and* $w^{\downarrow} = L_U$*. Else, continue to next step.*

4. **(Re-marking HtM I)** *If*

$$
\begin{aligned}
&L - x - (1 - \tfrac{1}{\lambda_{\max}})\ell \in [s\bar{f}(s) + (1 - \tfrac{1}{\lambda_{\max}})hf(s), (s + h)\bar{f}(s + h)], \quad \textit{and} \\
&L_U \geq \lambda_{\max} L - (\lambda_{\max} - 1)(x + \gamma^* \bar{f}(\gamma^*) + (s + h - \gamma^*)f(\gamma^*) + \ell), \quad \textit{for} \\
&\gamma^* \bar{f}(\gamma^*) + (1 - \tfrac{1}{\lambda_{\max}})(s + h - \gamma^*)f(\gamma^*) = L - x - (1 - \tfrac{1}{\lambda_{\max}})\ell, \quad \gamma^* \in [s, s + h],
\end{aligned}
$$

then $\gamma^{\downarrow} = \gamma^*$ *and* $w^{\downarrow} = x + \gamma^* \bar{f}(\gamma^*) \in (x, L_U)$*. Else, continue to next step.*

5. **(Re-marking HtM II)** *If* $L_U \in (x, x + (s + h)\bar{f}(s + h)]$ *and* $L_I \geq (1 - \frac{1}{\lambda_{\max}})[(s + h - \gamma^*)f(\gamma^*) + \ell]$ *for* $\gamma^* \in [s, s + h]$ *solving* $\gamma^* \bar{f}(\gamma^*) = L_U - x$*, then* $\gamma^{\downarrow} = \gamma^*$ *and* $w^{\downarrow} = L_U$*. Else, continue to next step.*

6. **(Illiquidity)** *If it gets to this final step, either*

$$
\begin{aligned}
&\lambda_{\max} L - (\lambda_{\max} - 1)(x + (s + h)\bar{f}(s + h) + \ell) \geq L_U \quad \textit{and} \quad L_U - x \geq (s + h)\bar{f}(s + h), \quad \textit{or} \\
&\lambda_{\max} L - (\lambda_{\max} - 1)(x + (s + h)\bar{f}(s + h) + \ell) < L_U \quad \textit{and} \quad L \geq x + (s + h)\bar{f}(s + h) + (1 - \tfrac{1}{\lambda_{\max}})\ell,
\end{aligned}
$$

in which case $\gamma^{\downarrow} = s + h$ *with* $w^{\downarrow} = L_U$ *or* $w^{\downarrow} = \lambda_{\max} L - (\lambda_{\max} - 1)(x + (s + h)\bar{f}(s + h) + \ell) \in (x, L_U)$*, respectively.*

When the algorithm terminates, one must additionally confirm that the candidate clearing solution leaves the bank solvent, i.e., that (5) *is satisfied. If the algorithm terminates before Step 6, but* (5) *is violated, then the bank is liquid but insolvent. If the algorithm terminates in Step 6 and* (5) *is violated, then the bank is both illiquid and insolvent.*

4 Case studies based on the Silicon Valley Bank

The case of SVB default is an insightful example of bank balance sheet vulnerabilities leading to bank runs and can be analysed using our framework. In the next sections we describe how different elements of the SVB default story correspond to features of our model (subsection 4.1). Moreover, we run simulations to illustrate some key drivers of the bank runs and to show effectiveness of some policy interventions, fostering insured deposit base and a prudent allocation of assets between HtM and AfS portfolios. A time series of SVB's balance sheet prior to the default in March 2023 allows us to analyse how the bank's vulnerabilities evolved. Extrapolating from this statement, the model can be used then as a monitoring tool of bank balance sheet vulnerabilities.

4.1 Balance sheet dynamics of SVB

In a nutshell, as a report from the Federal Reserve Board shows (FRB, 2023), the Silicon Valley Bank mismanaged its balance sheet growth caused by funding inflow from the cyclical technology and venture capital (VC) sectors. Notably, it was partly supported by a period of exceptionally low interest rates after the 2020 COVID-19 crisis. SVB invested those inflows of deposits in longer-term securities, i.e., held-to-maturity (HtM), government or agency-issued mortgage-backed securities (agency MBS). These securities are low risk from a credit perspective and provide a predictable return based on the interest rate at the time of purchase. However, in the monetary policy regime, the asset portfolios were not effectively managed from the interest-rate risk perspective. Notably, SVB was actively removing hedges as rates were rising. At the same time, SVB failed to manage the risks of its highly concentrated liabilities, which proved much more unstable than anticipated.

Changing market conditions led to cash-constrained tech and VC-backed firms which started to withdraw their deposit. The velocity of outflows was quickly accelerated as social networks, media, and other ties reinforced a run dynamic that played out at remarkable pace. SVB reached a point in March 2023 when it was forced to announce a restructuring of its balance sheet, including a completed sale of USD 21 billion of AFS securities for a USD 1.8 billion after-tax loss. Notably, the HtM accounting regime was constraining the bank from further raising cash as dipping into HtM securities would result in a reclassification of the whole HtM portfolio and booking unrealised HtM losses in SVB's profit and loss accounts.

These several factors, i.e., ailing management and governance, fragile business model, and changing market conditions combined to lead to a detrimental bank run. Which of them were the most impactful and which could be immunized to avert the collapse? Our model can be used to help address these questions. Given how parsimonious our framework is, we can use publicly available information about SVB to calibrate all crucial parameters of the model. Table 1 collects a time series of data characterizing the evolution of SVB. Between Q1 2020 and Q1 2022, i.e., one quarter before the collapse, total deposits grew more than threefold, from USD 56 billion to USD 181 billion. Only a small fraction of the funding base was insured deposits (USD 9 billion out of the USD 181 billion in Q1 2022). The absorbed funding was mostly invested into HtM securities (increase from USD 10 billion to USD 101 billion). When expectations about interest rate increases built, and eventually interest rated started to rapidly raise, the market value of the securities was gradually declining. However, thanks to the accounting treatment regarding how their value would be reflected in the financial results, this was only reflected in a build-up of the unrealized losses (increase from a gain of USD 0.8 billion to a loss of -15 USD billion in Q2 2022). This meant that even though the reported leverage ratio was hovering around 7.0 and 8.0 (measured by a ratio of total assets to Tier 1 capital), a leverage ratio factoring in the unrealized losses from HtM securities, and also from AfS portfolios, soared to almost 40.0. The described collection of balance sheet parameters of SVB is the main data source for the calibration of our model to run simulations to identify some tipping point parameters in the unwinding of a bank run on SVB.

4.2 Drivers of bank runs and policy implications

Based on the calibrated model we run simulations to show how leverage ratio targeting of depositors, fire sale price impact, unrealised losses and uncertainty of bank asset valuations impact bank run risk. We also show effectiveness of some policy interventions, related to the allocation of liabilities between insured and uninsured funding and of assets between available for sale and held for trading securities.

Target leverage ratio. By looking at the funding withdrawals (Figure 2) and asset liquidations in equilibrium, we can analyse the evolution of vulnerabilities of SVB balance sheet. We do not observe the targeted leverage ratio of the bank. However, we can infer its rough estimates from a relatively

		In USD billion										*Ratio*	
		Total deposits	Other funding	Insured deposits	Capital	Total assets	Cash	AfS	HtM	Unrealised Gains/Losses (HtM)	Unrealised Gains/Losses (AfS)	Tier 1 lev. ratio	Lev. ratio implied by Unrealised Gains/Losses
2020	q1	56	8.9	5	10.1	75	8	20	10	0.8	1.6	6.4	6.0
	q2	70	7.9	5	12.1	90	10	25	10	0.8	1.6	6.4	6.2
	q3	80	6.5	5	13.5	100	12	28	12	0.8	1.6	6.4	6.3
	q4	95	8.8	5	16.2	120	13	35	15	0.8	1.6	6.4	6.5
2021	q1	110	11.7	5	18.3	140	16	30	40	0.0	0.0	6.6	7.6
	q2	130	18.3	6	21.7	170	18	25	60	0.0	0.0	6.8	7.8
	q3	152	10.0	7	23.0	185	21	25	80	-0.5	0.0	7.0	8.2
	q4	172	16.9	8	26.1	215	23	27	103	-1.0	0.0	7.2	8.6
2022	q1	181	17.3	9	26.7	225	22	27	101	-7.5	-1.5	7.4	12.7
	q2	170	20.0	10	25.0	215	20	27	98	-11.5	-2.0	7.6	18.7
	q3	162	28.5	10	24.5	215	19	27	95	-16	-3.0	7.8	39.2
	q4	160	31.0	10	24.0	215	17	27	93	-15	-3.0	8.0	35.9

Table 1: Balance sheet evolution of the SVB
Numbers shown starting from the beginning of 2020 when the dynamics of assets and liabilities started to materially change. "Lev. ratio implied by Unrealized Gains/Losses" = [Total assets]/([Capital]-[Unrealised Gains/Losses (HtM)]-[Unrealised Gains/Losses (AfS)]); "Other funding" = calibrated such that balance sheet identity is preserved and leverage ratio reported by SVB ([Tier 1 ratio]) equals to the calculated leverage ratio (i.e., [Total assets]/[Capital]), "AfS" = securities in available for sale accounting portfolios; "HtM" = securities in held-to-maturity accounting portfolios
Source: SVB financial reports and FRB (2023)

stable period before 2021 when policy interest rates were low and the expectations of the hikes were still moderate, and SVB balance sheet did not start to balloon. The leverage ratio of SVB was hovering at around 7.0, computed as total assets divided by Tier 1 capital. Given the uncertainty about the exact leverage target, we consider a range of leverage ratios between 7.0 and 8.0. Until Q1 2021, funding withdrawals implied by the model are very limited and can be fully covered by cash holdings of SVB, depicted by the green bars. Only after, we can see rising equilibrium levels of funding withdrawals. Moreover, they start to imply runs necessitating liquidation of AfS portfolios. Only as of Q4 2022, the model indicated that runs following a higher leverage targeting could deprive SVB of available liquid resources and lead to dipping into HtM portfolios.

Fire-sale price impact. The other significant parameter of the model is the fire-sale impact of securities liquidations. This parameter is difficult to pin down (see Sydow et al. (2021)) and we conduct sensitivity analysis of our results to the price impact function. Figure 3 shows the amount of liquidated assets under various regimes of the price impact functions. Clearly, the more sensitive the valuation of assets to the sold volumes, the larger the needs to liquidate securities to restore liquidity.

Unrealised losses in HtM portfolios. The accounting of losses in securities portfolios masked the actual vulnerabilities stemming from the securities repricing pressure in the rising interest rate environment. However, the trigger for SVB's bankruptcy was related to investors expectations about growing hidden losses. From the hindsight, knowing the estimates of the unrealised losses (FRB, 2023), we can analyse how vulnerable the balance sheet of SVB was by assuming that the unrealised losses were to be reflected in capital and computing the implied withdrawals and liquidations in our simple model. To achieve that, we subtracted the estimated unrealised losses from AfS and HtM portfolios of SVB before running the simulations. Figure 4 shows that already at the the beginning of 2022 financial conditions of SVB became conducive to bankruptcy. In Q1, SVB would stay solvent but may already be considered illiquid and as of the subsequent periods, assuming a higher sensitivity of asset values in fire sales, the bank could be considered illiquid and insolvent. The outcomes of the simulations indicate that, given the mounting unrealised losses, the financial conditions of SVB would deteriorate sharply.

We can also align the vulnerabilities that picked up in 2022 with SVB's outlook for income presented in earnings reports. SVB was revising their net interest income upwards in Q4 2021 (30% growth year on year), to 50% in Q1 2022 that was attracting investors. However, Q2 2022 brought a downward revision of the net interest income to 40%, coinciding with the FED's aggressive monetary tightening policy. The reversal of the projected trend and related volatility made the financial situation of SVB very uncertain which is reflected in a sharp increase of the run risk, as measured by our model in Fig. 4. Admittedly, supervisors did not act timely and preemptively to prevent the ultimate run to happen.

Share of insured deposits. The model allows us to test some policy interventions that may reduce vulnerabilities in the balance sheets of banks. The most straightforward one in the case of SVB, advocated by researchers and policy makers in the wake of SVB meltdown, was to foster diversified or insured funding sources. We can directly test the impact of the later. To this end, we assume that a certain fraction of the uninsured deposits of SVB would be moved from uninsured to the insured category and, consequently, limiting the scope of the run *ex ante*. The Figure 5 shows results of the simulations. They show that reducing the volume of uninsured deposits by as much as 95% can eliminate the conditions for a run with adverse impact. However, limiting the size of uninsured funding by half, can already limit the size of financial losses, even though solvency default might not have been avoided. The banks' balance sheet was not sound enough to withstand the unhedged losses confirming that the adopted business model was flawed.

Allocation of securities to AfS and HtM porfolios. Since accounting of securities held by SVB was blamed for the collapse of the bank we can use the model to test whether a different allocation of securities across AfS and HtM portfolio could increase SVB's balance sheet soundness. To this end, we ran a simulation assuming that SVB would held more AfS securities. Practically, we reallocated a fraction of HtM portfolio to AfS assuming that the same amount of unrealised losses be incurred. We experimented with fractions ranging from 20% to 80%. Fig. 6 shows equilibrium withdrawal of deposits across time and different accounting structure of securities portfolios. The main finding is the following: the reallocation *per se* would not save the bank but rather delay a meltdown. In the critical period, i.e. Q1 2022, depicted by the results of our model, holding significantly more AfS securities would allow the bank to raise liquidity following some depositors decision to withdraw but not yet utilizing resources locked in the HtM portfolios. Notably, this result sheds light on the importance on interest rate risk management, including effective hedging, since the fair value depreciation reflecting market conditions is independent of accounting standards. HtM accounting hid the losses but not mitigated them and only a proper risk management could have been effective.

5 On the balance between HtM and AfS

To understand why banks hold HtM we should look at basic principles of banks' asset and liability management. Banks' business models – at least more traditional ones of the universal banks – rely on maturity transformation, i.e., banks invest in long-term projects financed by short-term funding. However, investment in long-term projects may be achieved via non-marketable loans or via bonds and equities, which are more liquid, frequently exchange-traded. However, because of liquidity needs, banks hold bonds also to be able to quickly raise cash, and holding period of those bonds may be short. Since these instruments are marked-to-market, they create volatility in banks' profit and loss accounts. To decrease the variability of income from bonds, and other securities, which are intentionally held for long-term investment purposes (i.e., held-to-maturity) regulators introduced accounting rules that allow banks to recognize these bonds at amortised costs. However, this designation is a commitment and whenever banks sell even a fraction of those bonds they need to derecognize the whole HtM portfolio. It is very penalizing and banks tend to keep some optimum level of the HtM securities, i.e., enough available for sale securities to cover liquidity needs under almost all foreseeable scenarios. Our model can help explain what the preferred level of HtM can be given funding risk of banks.

Specifically, consider a bank that holds A total assets and $\bar{A} := A - x - \ell$ of marketable securities

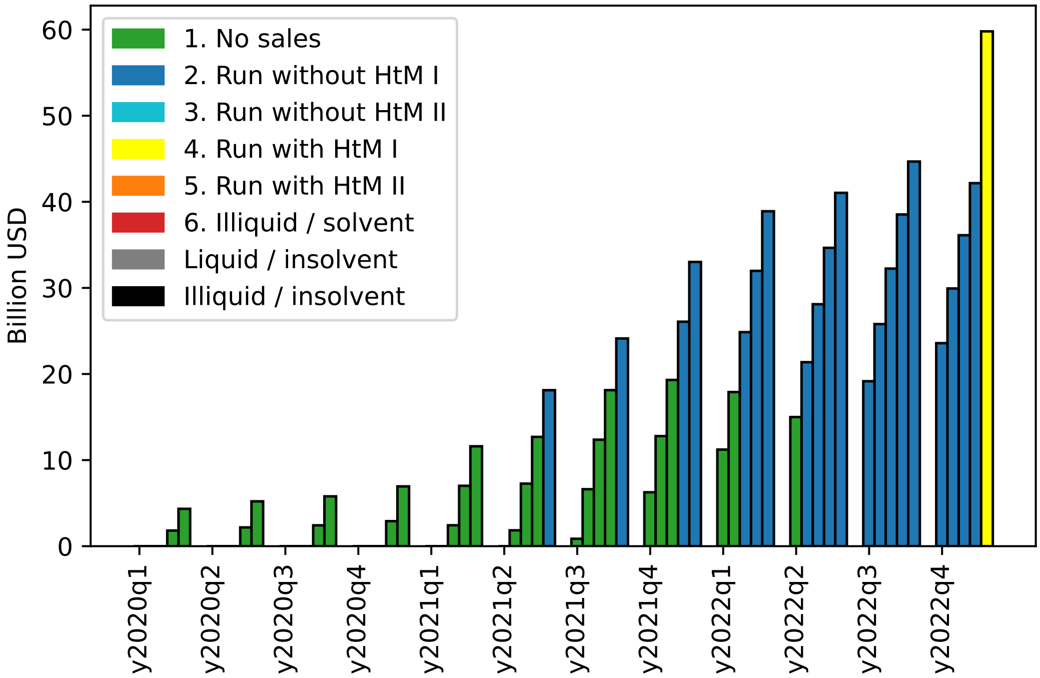

Figure 2: The figure shows equilibrium withdrawal of funding (in billion USD) from SVB for balance sheets observed between Q1 2020 and Q4 2022 and for various calibrations of targeted leverage ratios. For each period there is a group of bars, each of them corresponding to one leverage ratio from the set $\{7.0,\ 7.25,\ 7.5,\ 7.75,\ 8.0\}$. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

that can be designated as AfS or HtM, i.e., so that $\bar{A} = s + h$. Herein, implicitly, we consider the setting in which the initial price $p = 1$ at the time that the bank determines the composition of its balance sheet. However, we assume the price p of these securities fluctuates randomly (e.g., following a lognormal distribution), which generates risks on the banking book. Such risks are evident from the failure of SVB in which the price of its HtM securities fell on the open market causing unrealized losses.

To simplify the setting, we consider the bank to want as much of its marketable securities held to maturity as possible, while keeping the probability that it needs to remark those assets below $\alpha \in (0, 1)$. That is, the bank seeks to optimize

$$\begin{aligned} h^* &= \max\{h \in [0, \bar{A}] \mid \gamma^{\downarrow}(p_\alpha) \leq \bar{A} - h\} \\ &= \bar{A} - \min\{s \in [0, \bar{A}] \mid \gamma^{\downarrow}(p_\alpha) \leq s\} \end{aligned} \tag{6}$$

where the dependence of the minimal liquidations on the initial price is made explicit and $p_\alpha > 0$ is the α-lower quantile of the price distribution.

Proposition 5.1 (Optimal balance sheet). *Assume $\lambda_{\max} \geq 2$. Assume the linear inverse demand function $f_\alpha(\gamma) = p_\alpha(1 - b\gamma)$ with $b < 1/[(\lambda_{\max} - 1)\bar{A}]$. The minimal AfS $s^* = \min\{NS, PW, FW\}$ where*

$$NS = \begin{cases} 0 & \text{if } L_U > x \text{ or } L \leq x + \frac{\lambda_{\max}-1}{\lambda_{\max}}(\bar{A} + \ell) \\ \bar{A} & \text{else} \end{cases}$$

$$PW = \begin{cases} \frac{p_\alpha - \frac{\lambda_{\max}-1}{\lambda_{\max}} - M}{bp_\alpha} & \text{if } \frac{p_\alpha - \frac{\lambda_{\max}-1}{\lambda_{\max}} - M}{bp_\alpha} \leq \bar{s} \text{ and } p_\alpha \geq \frac{\lambda_{\max}-1}{\lambda_{\max}} + b[L - x - \frac{\lambda_{\max}-1}{\lambda_{\max}}(\bar{A} + \ell)] + M \\ \bar{A} & \text{else} \end{cases}$$

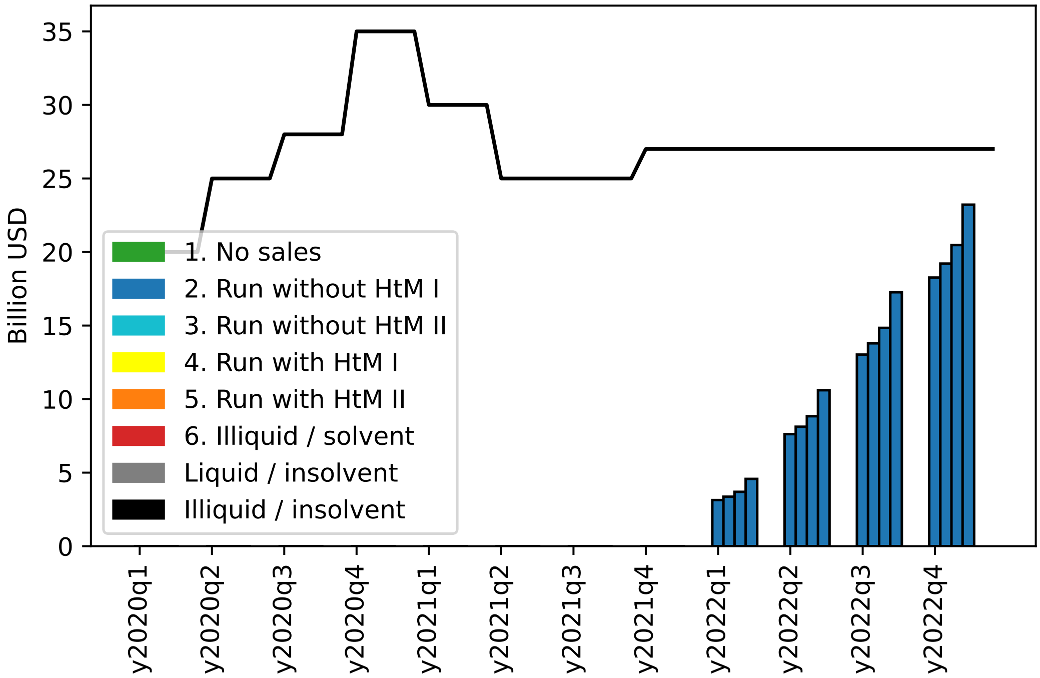

Figure 3: The figure shows equilibrium liquidation of securities by SVB (in billion USD) for balance sheets observed between Q1 2020 and Q4 2022 and for various parameterisations of price impact functions. For each period there is a group of bars, each of them corresponding to one parameter of the linear impact function (b) from the set $\{0.0001, 0.0002, 0.001, 0.002\}$. For instance, 0.0001 corresponds to 10 bp impact on asset prices when 10 billion USD of securities are liquidated, like in Greenwood et al. (2015). Target leverage ratio = 7.5. Black line indicated the total volume of securities in the AfS portfolio. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

$$\textit{where } M = \sqrt{(p_\alpha - \frac{\lambda_{\max}-1}{\lambda_{\max}})^2 - 2p_\alpha b[L - x - \frac{\lambda_{\max}-1}{\lambda_{\max}}(\bar{A}+\ell)]},$$

$$FW = \begin{cases} \max\{\gamma^*\ ,\ \frac{\bar{A}+\ell-p_\alpha\gamma^*(1-b\gamma^*)-\frac{\lambda_{\max}}{\lambda_{\max}-1}L_I}{1-p_\alpha(1-b\gamma^*)}\} & \textit{if } p_\alpha \geq 2b(L_U - x) \\ \bar{A} & \textit{else} \end{cases}$$

and $\gamma^* = \frac{1-\sqrt{1-2b\frac{L_U-x}{p_\alpha}}}{b}$ *and with* $\bar{s}$ *defined as the solution to*

$$L_u = \lambda_{\max}L - (\lambda_{\max}-1)[x + \bar{\gamma}(\bar{s})f(\bar{\gamma}(\bar{s})) + (\bar{s}-\bar{\gamma}(\bar{s}))f(\bar{\gamma}(\bar{s})) + \bar{A} + \ell - \bar{s}]$$

for $(\bar{l} = \frac{\lambda_{\max}-1}{\lambda_{\max}})$

$$\bar{\gamma}(s) := \frac{-p_\alpha(1-b(\lambda_{\max}-1)s) + \sqrt{[\frac{p_\alpha}{\lambda_{\max}}(1-b(\lambda_{\max}-1)s)]^2 - 4p_\alpha(\bar{l}-\frac{1}{2})b[(1+p_\alpha)\bar{l}s - (L-x-\bar{l}(\bar{A}+\ell))]}}{p_\alpha(\lambda_{\max}-2)b}.$$

Proof. There are three steps of the clearing algorithm in Proposition 3.2 before HtM is re-marked: (i) no sales; (ii) run without re-marking HtM I; and (iii) run without re-marking HtM II. To determine the minimal AfS, we will determine the feasible AfS so that we end up in each of these cases.

(i) The no sales case are feasible if either $L_U \leq x$ or $L \leq x + \frac{\lambda_{\max}-1}{\lambda_{\max}}(\bar{A}+\ell)$. As these conditions do not depend on the decomposition of $\bar{A}$ into AfS and HtM, either any balance sheet is feasible or none is. The minimal possible value in this feasible region is, thus, given by NS.

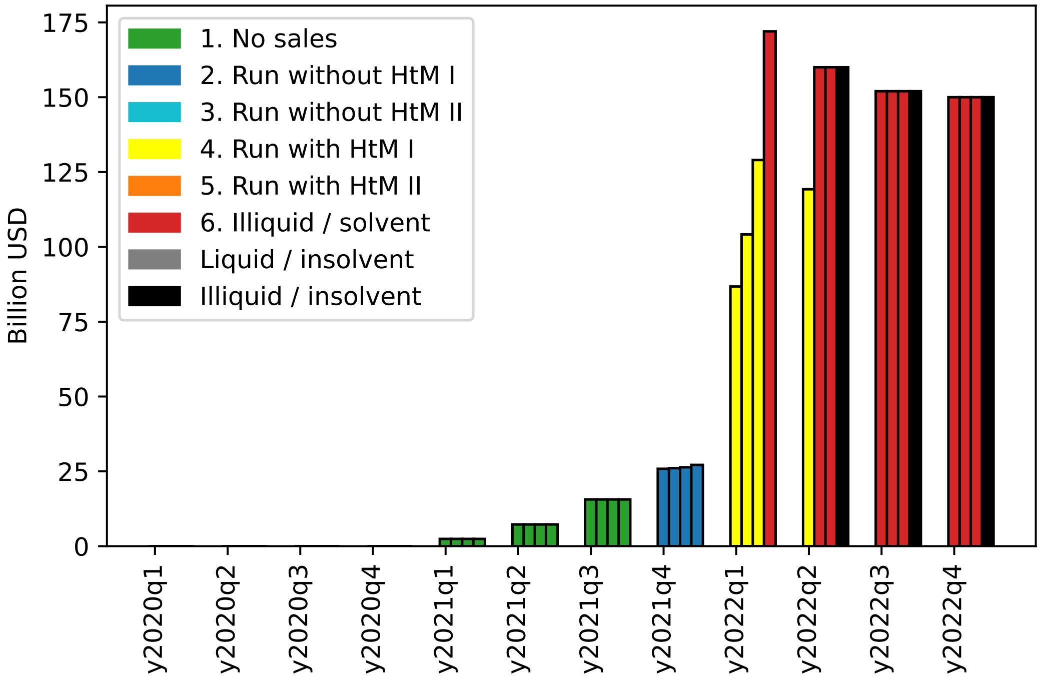

Figure 4: The figure shows equilibrium funding withdrawals from SVB (in billion USD) in a hypothetical scenario of unrealised losses in AfS and HtM portfolios being realized in the value of the securities portfolios and for balance sheets observed between Q1 2020, and Q4 2022 and for various parameterisations of price impact functions. For each period there is a group of bars, each of them corresponding to one parameter of the linear impact function (b) from the set $\{0.0001,\ 0.0002,\ 0.001,\ 0.002\}$. For instance, 0.0001 corresponds to 10 bp impact on asset prices when 10 billion USD of securities are liquidated, like in Greenwood et al. (2015). Target leverage ratio = 7.5. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

(ii) Let $\bar{\gamma}, \bar{s}$ be given as in the statement of the proposition. [4] Then the run without re-marking HtM I case is feasible if and only if $L - x - \frac{\lambda_{\max}-1}{\lambda_{\max}}(\bar{A} - s + \ell) \leq s\bar{f}(s)$ (the lower bound for which is provided within PW) and $L_U \geq \lambda_{\max} L - (\lambda_{\max} - 1)(x + \bar{\gamma}\bar{f}(\bar{\gamma}) + (s - \bar{\gamma})f(\bar{\gamma}) + \bar{A} - s + \ell)$, i.e., $s \leq \bar{s}$. The minimal possible value in this feasible region is therefore given by PW.

(iii) Let γ^* be such that $\gamma^* \bar{f}(\gamma^*) = L_U - x$, i.e., as given in the statement of the proposition (which exists if and only if $p_\alpha \geq 2b(L_U - x)$). Then the run without re-marking HtM II case is feasible if and only if $s \geq \gamma^*$ (so that $s\bar{f}(s) \geq L_U - x$) and $L_I \geq \frac{\lambda_{\max}-1}{\lambda_{\max}}[(s - \gamma^*)f(\gamma^*) + \bar{A} - s + \ell]$. The minimal possible value in this feasible region is given by FW.

□

A Proofs

A.1 Proof of Proposition 3.1

One readily confirms that the two mappings (2) and (3) are non-decreasing in $(w, \gamma) \in [0, L_U] \times [0, s+h]$. As the domain of Φ defined by (2)- (3) is a complete lattice, the claim follows from Tarski's fixed point theorem.

[4]It has to be ensured that $\bar{\gamma}$ exists and $\bar{s}$ unique solution.

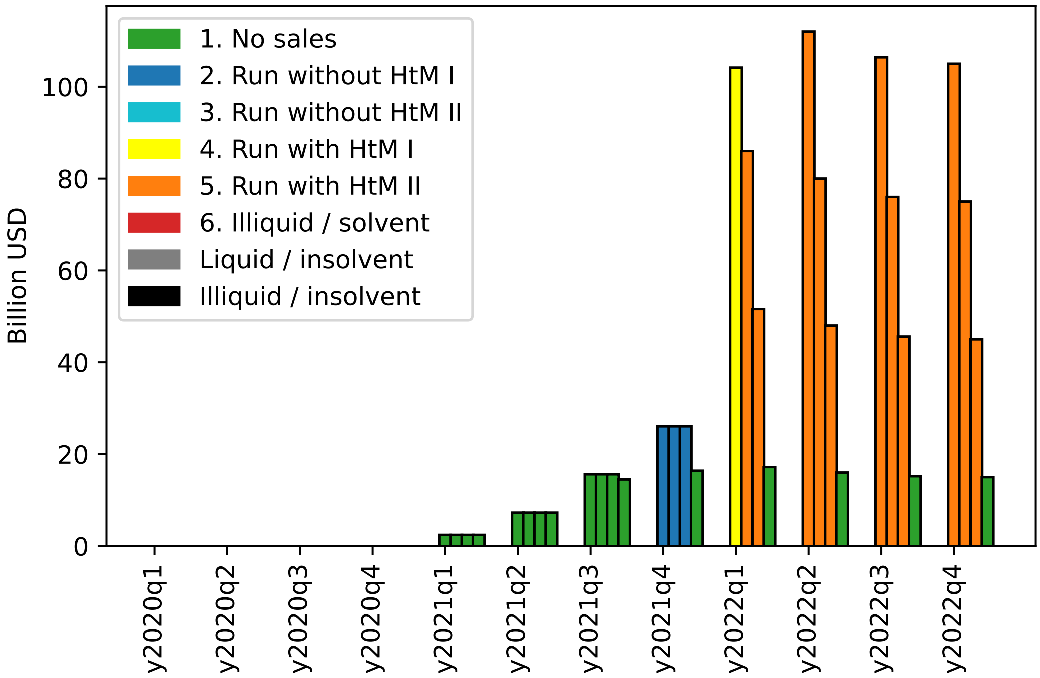

Figure 5: The figure shows equilibrium funding withdrawals from SVB (in billion USD) in a hypothetical scenario of unrealised losses in AfS and HtM portfolios being realized in the value of the securities portfolios and *ex ante* policy interventions limiting the size of the uninsured deposits. Results are shown for balance sheets observed between Q1 2020, and Q4 2022 and for various parameterisations of price impact functions. For each period there is a group of bars, each of them corresponding to one parameter of the reduction in the volume of uninsured deposits taken from the set $\{40\%,\ 55\%,\ 70\%,\ 95\%\}$. For instance, 40% means that 40% of uninsured deposits are moved to insured deposits category. Target leverage ratio = 7.5. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

A.2 Proof of Proposition 3.2

By Proposition 3.1 there exists a minimal clearing solution $(w^*, \gamma^*) \in [0, L_U] \times [0, s+h]$, provided the bank is solvent. The left-hand side of the solvency condition (5) reads as

$$x + \gamma \bar{f}(\gamma) + (s-\gamma) f(\gamma) + h + \ell, \quad \text{for} \quad \gamma \in [0, s], \quad \text{and}$$

$$x + \gamma \bar{f}(\gamma) + (s+h-\gamma) f(\gamma) + \ell, \quad \text{for} \quad \gamma \in (s, s+h].$$

Following Remark 2.7, these are both non-increasing functions of γ on the respective domains. Moreover, at $\gamma = 1$, there is a jump of size $(f(s)-1)h \leq 0$, since $f(s) \leq 1$ by Assumption 2.2. Consequently, if the bank was already insolvent at some level of liquidations γ, it is also insolvent for all larger values. It therefore suffices to check for solvency at the termination of the algorithm, since the algorithm is increasing in the value of γ^*.

By construction, we must have that either $\gamma^* = 0$ (no sales), $\gamma^* \in (0, s]$ (run without re-marking of HtM), $\gamma^* \in (s, s+h)$ (run with re-marking of HtM), or $\gamma^* = s+h$ (illiquidity). By studying these case-by-case, we will be able to conclude that our clearing solution is indeed realized by one of the steps presented in Proposition 3.1. By proceeding in increasing order with respect to the values of (γ^*, w^*), we arrive at the minimal solution.

Step 1 (No sales). Assume $\gamma^* = 0$. Then $w^* = \Phi_w(0) = L_U \wedge [\lambda_{\max} L - (\lambda_{\max}-1)(x+sp+h+\ell)]^+$. This is a clearing solution if and only if $w^* \leq x$. This, in turn, holds if and only if $L_U \leq x$ or

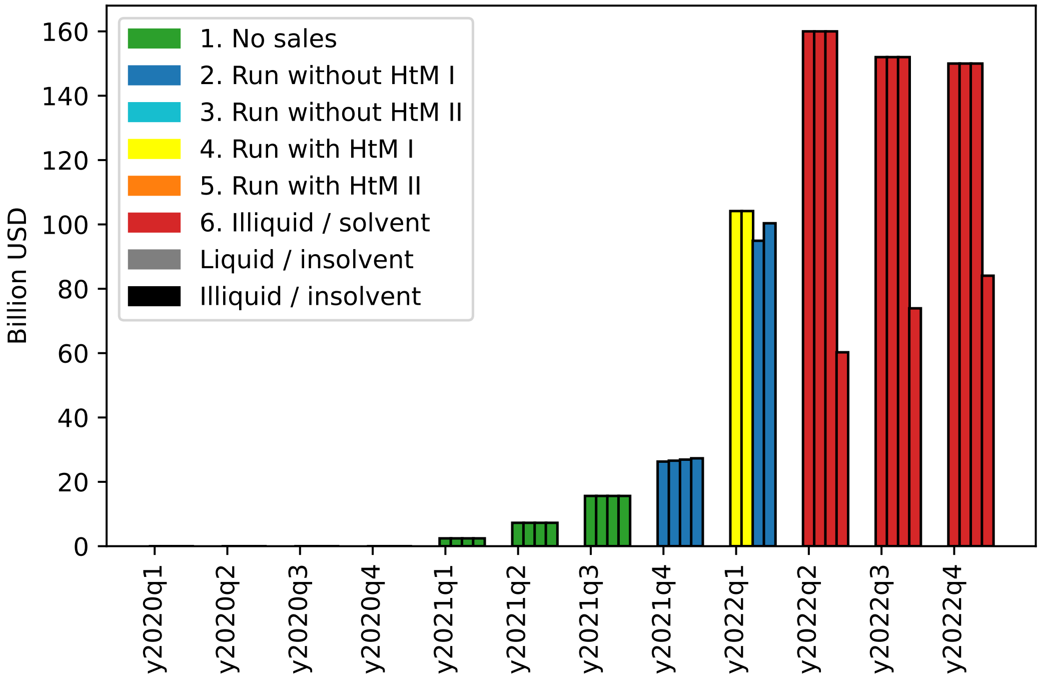

Figure 6: The figure shows equilibrium funding withdrawals from SVB (in billion USD) in a hypothetical scenario of unrealised losses in AfS and HtM portfolios being realized in the value of the securities portfolios and *ex ante* different allocation of securities to AfS and HtM accounting portfolios. Results are shown for balance sheets observed between Q1 2020, and Q4 2022 and for various parameterisations of price impact functions. For each period there is a group of bars, each of them corresponding to one parameter of the percentage reduction in the volume of HtM securities and allocating to the AfS portfolio. Reduction parameters are selected from the set {20%, 40%, 60%, 80%}. For instance, 40% means that 40% of HtM securities are moved to the AfS category. Target leverage ratio = 7.5. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

$\lambda_{\max}L - (\lambda_{\max} - 1)(x + sp + h + \ell) \leq x$. The latter holds if only if

$$L \leq \frac{x}{\lambda_{\max}} + \frac{\lambda_{\max} - 1}{\lambda_{\max}}(x + sp + h + \ell) = x + \frac{\lambda_{\max} - 1}{\lambda_{\max}}[sp + h + \ell].$$

Step 2 (Run without re-marking HtM I). Suppose $\gamma^* \in (0, s]$. Then $w^* \in (x, L_U]$ with $L_U > x$. For this step, assume $w^* \in (x, L_U)$. Since $\gamma^* \in (0, s]$, we see that we$w^* = \Phi_w(\gamma^*)$ holds if and only if

$$L_U \geq \lambda_{\max}L - (\lambda_{\max} - 1)(x + \gamma^* \bar{f}(\gamma^*) + (s - \gamma^*)f(\gamma^*) + h + \ell). \tag{7}$$

Note that w^* equals the right-hand side of (14). Moreover, γ^* must satisfy $\gamma^* \bar{f}(\gamma^*) = w^* - x$, and it is the unique such solution, since the left-hand side is strictly increasing. Inserting $w^* = x + \gamma^* \bar{f}(\gamma^*)$ in (14) and recalling that the right-hand side equals w^*, we obtain

$$w^* = L - (1 - \frac{1}{\lambda_{\max}})((s - \gamma^*)f(\gamma^*) + h + \ell).$$

Thus, the liquidation $\gamma^* \in (0, s]$ satisfies

$$\gamma^* \bar{f}(\gamma^*) + (1 - \frac{1}{\lambda_{\max}})(s - \gamma^*)f(\gamma^*) = L - x - (1 - \frac{1}{\lambda_{\max}})(h + \ell), \tag{8}$$

and it must be the unique solution to this equation on $(0, s]$, since the left-hand side is strictly increasing in γ^* on $[0, s]$ by Assumption 2.5 and Remark 2.7. This is possible if and only if

$$L - x - (1 - \frac{1}{\lambda_{\max}})(h + \ell) \in \big[(1 - \frac{1}{\lambda_{\max}})sp, s\bar{f}(s)\big]. \tag{9}$$

Consequently, we have a clearing solution if and only if both (9) and (14) hold with γ^* in (14) being the unique solution to (8).

Step 3 (Run without re-marking HtM I). Now assume $\gamma^* \in (0, s]$ and $w^* = L_U$. Then γ^* satisfies $\gamma^* \bar{f}(\gamma^*) = w^* - x = L_U - x$. As the left-hand side is strictly increasing, we have a unique solution which is in $(0, s]$ if and only if $L_U \in (x, x + s\bar{f}(s)]$. With $\gamma^* \le s$, we have $w^* = \Phi_w(\gamma^*) = L_U$ if and only if $L_U \le \lambda_{\max} L - (\lambda_{\max} - 1)(x + \gamma^* \bar{f}(\gamma^*) + (s - \gamma^*)f(\gamma^*) + h + \ell)$. Writing $L = L_I + L_U$, this re-arranges to

$$L_I \ge (1 - \frac{1}{\lambda_{\max}})[(s - \gamma^*)f(\gamma^*) + h + \ell]. \tag{10}$$

Consequently, (γ^*, w^*) is a clearing solution if and only if $L_U \in (x, x + s\bar{f}(s)]$ and (10) holds for the unique solution $\gamma^* \in (0, s]$ of $\gamma^* \bar{f}(\gamma^*) = L_U - x$.

Step 4 (Re-marking HtM I). Suppose $\gamma^* \in (s, s+h)$. Then $w^* \in (x, L_U]$. For this step we assume $w^* \in (x, L_U)$. We have $w^* = \Phi^*_w(\gamma^*)$ if and only if

$$L_U \ge \lambda_{\max} L - (\lambda_{\max} - 1)(x + \gamma^* \bar{f}(\gamma^*) + (s + h - \gamma^*)f(\gamma^*) + \ell), \tag{11}$$

and w^* is then given by the right-hand side of (11). Noting that γ^* must be the unique solution of $\gamma^* \bar{f}(\gamma^*) = w^* - x$, we can insert this in (11) and solve for

$$w^* = L - (1 - \frac{1}{\lambda_{\max}})((s + h - \gamma^*)f(\gamma^*) + \ell).$$

In turn, $\gamma^* \in (s, s + h)$ must solve

$$\gamma^* \bar{f}(\gamma^*) + (1 - \frac{1}{\lambda_{\max}})(s + h - \gamma^*)f(\gamma^*) = L - x - (1 - \frac{1}{\lambda_{\max}})\ell, \tag{12}$$

and it must the unique such solution since the left-hand side is strictly increasing on $[0, s + h]$ by Assumption 2.5. This is feasible if and only if

$$L - x - (1 - \frac{1}{\lambda_{\max}})\ell \in \big[s\bar{f}(s) + (1 - \frac{1}{\lambda_{\max}})hf(s), (s + h)\bar{f}(s + h)\big]. \tag{13}$$

In conclusion, we have a clearing solution if and only if (13) and (11) hold, when γ^* in (11) is given by the unique solution to (12).

Step 5 (Re-marking HtM II). Now assume $\gamma^* \in (s, s+h)$ and $w^* = L_U$. Then $\gamma^* \bar{f}(\gamma^*) = L_U - x$, which is possible if and only if $L_U \in (x, x + (s + h)\bar{f}(s + h))$. Moreover, we see that $\Phi_w(\gamma^*) = L_U$ holds if and only if

$$L_I \ge (1 - \frac{1}{\lambda_{\max}})[(s + h - \gamma^*)f(\gamma^*) + \ell]. \tag{14}$$

We thus have a clearing solution if and only if $L_U \in (x, x + (s + h)\bar{f}(s + h))$ and the unique solution γ^* to $\gamma^* \bar{f}(\gamma^*) = L_U - x$ satisfies (14).

Step 6 (Illiquidity). Finally, assume $\gamma^* = s + h$. Then

$$w^* = \Phi_w(s + h) = L_U \wedge [\lambda_{\max} L - (\lambda_{\max} - 1)(x + (s + h)\bar{f}(s + h) + \ell)].$$

This is a clearing solution if and only if $w^* - x \geq (s+h)\bar{f}(s+h)$. Given the expression for w^*, this holds if and only if either

$$\lambda_{\max}L - (\lambda_{\max}-1)(x+(s+h)\bar{f}(s+h)+\ell) \geq L_U \quad \text{and} \quad L_U \geq x+(s+h)\bar{f}(s+h), \quad \text{or}$$
$$\lambda_{\max}L - (\lambda_{\max}-1)(x+(s+h)\bar{f}(s+h)+\ell) < L_U \quad \text{and} \quad L \geq x+(s+h)\bar{f}(s+h)+(1-\frac{1}{\lambda_{\max}})\ell,$$

since the last inequality is equivalent to

$$\lambda_{\max}L - (\lambda_{\max}-1)\big(x+(s+h)\bar{f}(s+h)+\ell\big) \geq x+(s+h)\bar{f}(s+h).$$

This completes the proof.

B Additional Sensitivity Analysis

Proposition 3.2 shows that there are some critical parameters of the model that change the liquidity and solvency state of the bank. One of them is the share of uninsured deposits that may be subject to a run risk. The other one is the uncertainty of the value of the assets held by the bank that impacts depositors' beliefs about the pool of assets that can be used to raise cash and cover deposit withdrawals. Notably, the market value of the balance sheet, even independent of the unrealised loss aspect, can be lower than the book value.

As Fig. 7 shows, the level of uninsured deposits influences the liquidity and solvency state of a bank. Moreover, a small shift in the composition of liabilities may determine whether the banks is liquid and solvent, being able to raise cash to cover potential bank run and hold adequate level of capital to cover even unexpected losses, or may lose solvency despite retaining capacity to satisfy immediate deposit withdrawals. This sheds light on the importance of careful calibration of requirements regarding banks' composition of funding sources.

We illustrated the impact of the initial valuation on the run risk in Fig. 8. It is interesting to see changes in the initial valuation of the asset portfolios can create jumps in the vulnerability of the banks. The impact is most visible at the end of 2022 when low valuation of securities portfolios may create conditions for deposit runs leading to illiquidity. The state of the bank is very sensitive to the beliefs regarding the valuation. For instance, as of Q4 2022, depending whether depositors believe 7.5% or 10% lower valuation of assets, the bank may move from liquid state, even though requiring tapping liquidity from HtM portfolios, to illiquidity meaning that the bank does not have enough resources to satisfy depositors.

References

Ahnert, T., Anand, K. and König, P. J. (2023), 'Real interest rates, bank borrowing, and fragility', *Journal of Money, Credit and Banking* **forthcoming**(n/a).

Altermatt, L., van Buggenum, H. and Voellmy, L. (2022), Systemic bank runs: How a misallocation of liquidity may trigger a solvency crisis, Policy Brief 490, SUERF.

Banerjee, T. and Feinstein, Z. (2021), 'Price mediated contagion through capital ratio requirements with VWAP liquidation prices', *European Journal of Operational Research* **295**(3), 1147–1160.

Bischof, J., Laux, C. and Leuz, C. (2021), 'Accounting for financial stability: Bank disclosure and loss recognition in the financial crisis', *Journal of Financial Economics* **141**(3), 1188–1217.

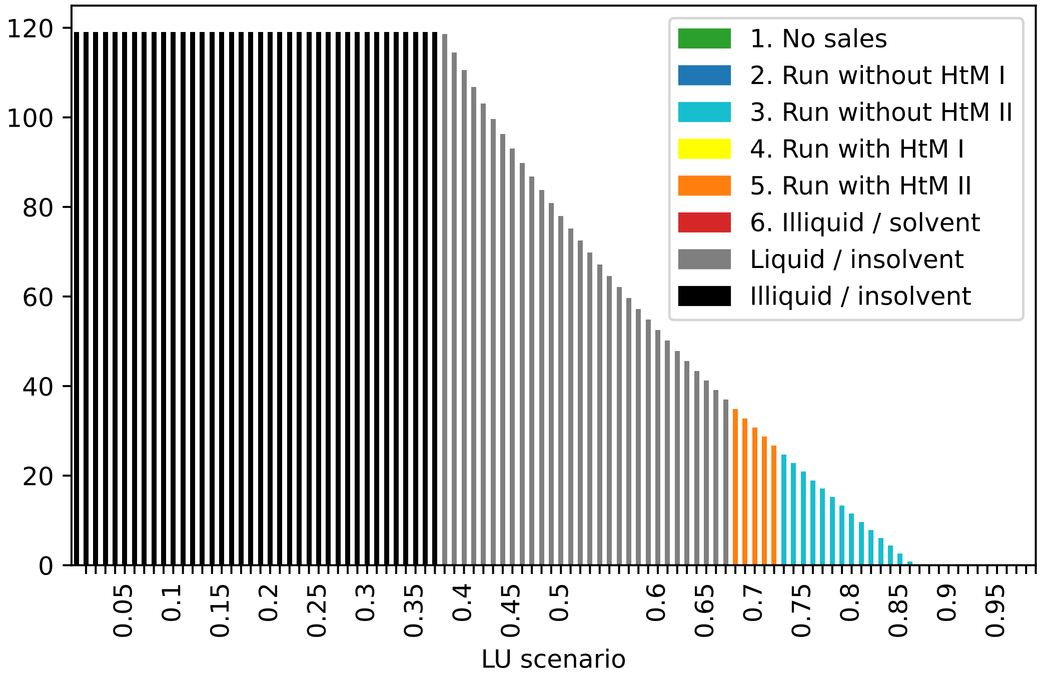

Figure 7: The figure shows equilibrium liquidation of assets (in billion USD) from SVB for balance sheets observed as of Q1 2022 for various calibrations of the share of uninsured deposits being converted into insured funding. Each bar corresponds to one value of a fraction of L_U, with the fractions varying from 0.0 to 1.0 (x-axis). For instance, a bar corresponding to 0.7 means that in case 70% of uninsured deposits are replaced by L_I category funding, the bank would solvent and liquid following funding withdrawals in equilibrium but would need to dip into HtM portfolios (orange bar) to generate enough cash. The target leverage ratio is set to 7.5. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

Cifuentes, R., Ferrucci, G. and Shin, H. (2005), 'Liquidity risk and contagion', *Journal of the European Economic Association* **3**(2/3), 556–566.

Diamond, D. W. and Dybvig, P. H. (1983), 'Bank Runs, Deposit Insurance, and Liquidity', *Journal of Political Economy* **91**(3), 401–419.

Diamond, D. W. and Rajan, R. G. (2011), ' Fear of Fire Sales, Illiquidity Seeking, and Credit Freezes', *The Quarterly Journal of Economics* **126**(2), 557–591.

Drechsler, I., Savov, A., Schnabl, P. and Wang, O. (2023), 'Banking on uninsured deposits', *NBER Working Paper* (31138).

Duca, M. L. and Peltonen, T. A. (2013), 'Assessing systemic risks and predicting systemic events', *Journal of Banking & Finance* **37**(7), 2183–2195.
URL: *https://www.sciencedirect.com/science/article/pii/S0378426612001628*

FRB (2023), Review of the Federal Reserve's Supervision and Regulation of Silicon Valley Bank, Guidance and Supervision, Federal Reserve Board.

Gertler, M. and Kiyotaki, N. (2015), 'Banking, liquidity, and bank runs in an infinite horizon economy', *American Economic Review* **105**(7), 2011–43.

Granja, J. (2023), Bank fragility and reclassification of securities into HTM, Working Paper 4409834, SSRN.

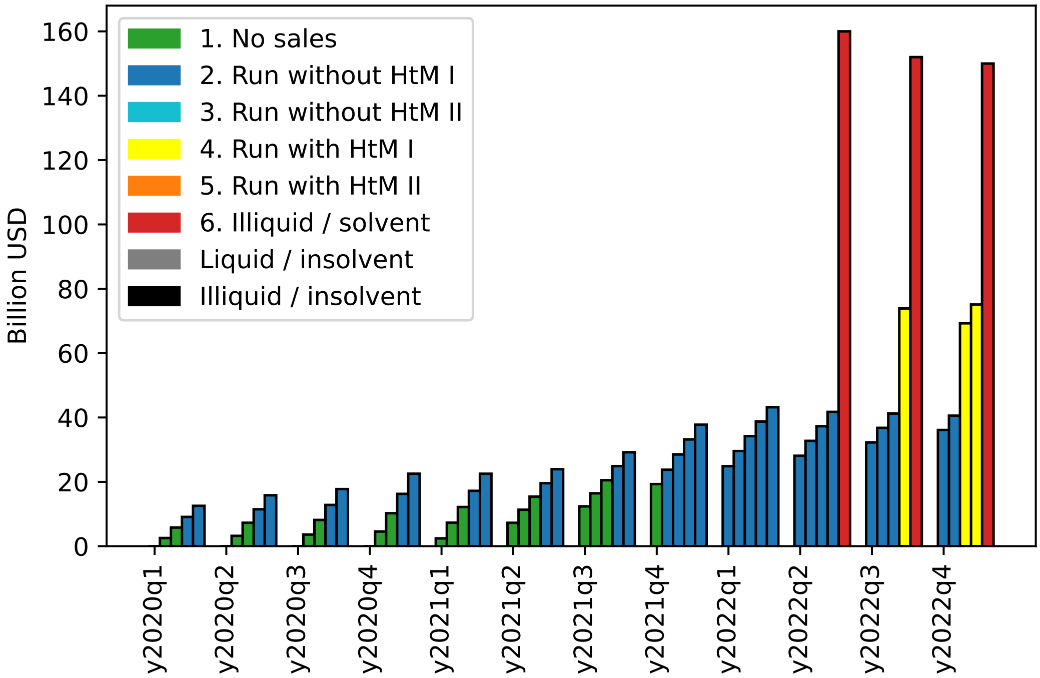

Figure 8: The figure shows equilibrium withdrawal of funding (in billion USD) from SVB for balance sheets observed between Q1 2020 and Q4 2022 and for various calibrations of initial valuation of marketable assets (i.e., parameter p). For each period there is a group of bars, each of them corresponding to one value of p from a set $\{1.000, \ , 0.975, \ , 0.950, \ 0.925, \ 0.900\}$ and target leverage ratio 7.5. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

Greenwood, R., Landier, A. and Thesmar, D. (2015), 'Vulnerable banks', *Journal of Financial Economics* **115**(3), 471–485.

Kim, S., Kim, S. and Ryan, S. G. (2023), Banks' motivations for designating securities as held to maturity, Working Paper 4452667, SSRN.

Liu, X. (2023), 'A model of systemic bank runs', *The Journal of Finance* **78**(2), 731–793.

Morris, S. and Shin, H. (2003), *Global games: Theory and applications*, Cambridge University Press, United Kingdom, pp. 56–114. Publisher Copyright: © Mathias Dewatripont, Lars Peter Hansen, and Stephen J. Turnovsky 2003 and Cambridge University Press, 2009.

Sydow, M., Schilte, A., Covi, G., Deipenbrock, M., Del Vecchio, L., Fiedor, P., Fukker, G., Gehrend, M., Gourdel, R., Grassi, A., Hilberg, B. and Ka (2021), Shock amplification in an interconnected financial system of banks and investment funds, Working Paper Series 2581, European Central Bank.
URL: *https://ideas.repec.org/p/ecb/ecbwps/20212581.html*

Uhlig, H. (2010), 'A model of a systemic bank run', *Journal of Monetary Economics* **57**(1), 78–96.