

Parallel Currencies in an NK Framework

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Motivation and Research Question

- ▶ Increasing varieties of privately issued digital currencies:
 - ▶ Bitcoin
 - ▶ Ethereum
 - ▶ ...
- ▶ **Question:** What happens, when firms price in these currencies, rather than the official currency?
- ▶ Role of money:
 1. **Unit of account.** Here: currency of pricing.
 2. Medium of exchange.
 3. Store of value.
- ▶ **Approach:** an NK model with multiple currencies.

Results Overview

- ▶ Relative price between sectors becomes state variable. Rich sectoral dynamics.
- ▶ In response to a dollar depreciation:
 - ▶ Considerable persistent reallocation between sectors. Large decline in non-dollar sector. Small and temporary aggregate recession.
 - ▶ Recession is persistent, if mon pol only reacts to dollar inflation.
 - ▶ Increased flexibility of prices in non-dollar sector mitigates output drop in that sector and sectoral reallocation. None at flexible limit.
 - ▶ Larger non-dollar sector share induces deeper overall recession, higher inflation, larger gain to dollar sector.
- ▶ Endogenous currency choice: less flexible sector increases substantially. Slight, persistent aggregate boom, inflation drop.

Literature

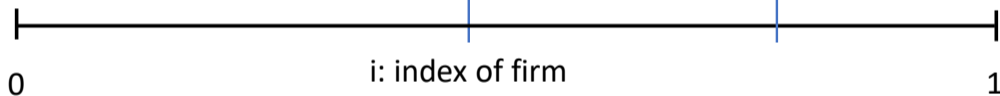
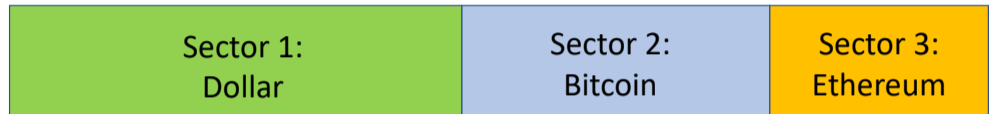
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- ▶ Cienfuegos, N. C. (2019, January). The Importance of Production Networks and Sectoral Heterogeneity for Monetary Policy. Technical report. University of Chicago.
- ▶ Schilling, L. and H. Uhlig (2019, October). Some simple bitcoin economics. *Journal of Monetary Economics* 106, 16-26.
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Model – Currencies and Prices

- ▶ J currencies in total, each with money supply $M_{j,t}$
 - ▶ $j = 1$: fiat currency, *dollar*; $j \neq 1$ parallel currency, *bitcoin*
 - ▶ $\mathcal{E}_{j,t}$: price of currency j in dollar
 - ▶ $\mathcal{E}_{1,t} = 1$
 - ▶ $\frac{\mathcal{E}_{j,t}}{\mathcal{E}_{j',t}}$: price of currency j in currency j'
- ▶ Firms in sector j set prices in currency j
 - ▶ $V_{j,t}$: set of firms in sector j
 - ▶ $v_{j,t}$: measure of sector j
 - ▶ sectoral price index $P_{j,t} = \left[\frac{1}{v_{j,t}} \int_{V_{j,t}} P_{j,t}(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}$
 - ▶ general price index $P_t = \left[\sum_{j=1}^J v_{j,t} (\mathcal{E}_{j,t} P_{j,t})^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$
 - ▶ general price inflation $\Pi_t = \frac{P_t}{P_{t-1}}$
 - ▶ sectoral relative price $\hat{P}_{j,t} = \frac{\mathcal{E}_{j,t} P_{j,t}}{P_t}$

Pricing Sectors

Firm i pricing currency:



Benchmark: Sectors are fixed

Extension: Endogenous
currency choice



Households

- ▶ Lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t, N_t)$$

- ▶ Consumption bundle $C_t = \left[\int C_t(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$

- ▶ Liquidity $L_t = \sum_{j=1}^J$ where $L_{j,t} = \frac{\mathcal{E}_{j,t} M_{j,t}}{P_t}$

- ▶ Labour supply N_t

- ▶ Budget constraint

$$C_t + \frac{B_t}{P_t} + \sum_{j=1}^J L_{j,t} = \frac{\exp(i_{t-1}) B_{t-1}}{\Pi_t P_{t-1}} + \sum_{j=1}^J \frac{L_{j,t-1}}{\Pi_t} \frac{\mathcal{E}_{j,t}}{\mathcal{E}_{j,t-1}} + W_t N_t + \Gamma_t$$

Firms

- ▶ Production function $Y_t(i) = A_t N_t(i)^{1-\alpha}$
- ▶ $1 - \theta_j$ fraction of firms reset prices in sector j
- ▶ Profit maximization problem

$$\max_{P_{j,t}^*} \sum_{l=0}^{\infty} \theta_j^l E_t \left[Q_{t,t+l} \left[\frac{\mathcal{E}_{j,t+l} P_{j,t}^*}{P_{t+l}} Y_{t+l}(i) - \Psi_{t+l}(Y_{t+l}(i)) \right] \right]$$

subject to demand function $Y_{t+l}(i) = \left(\frac{\mathcal{E}_{j,t+l} P_{j,t}^*}{P_{t+l}} \right)^{-\epsilon} Y_{t+l}$

Linearised Model

- ▶ **Proposition 1:** The nominal exchange rate between any pair of parallel currencies j and j' follows a random-walk process:

$$e_{j,t} - e_{j',t} = E_t (e_{j,t+1} - e_{j',t+1})$$

- ▶ Sectoral NKPC:

$$\pi_{j,t} = \beta E_t \pi_{j,t+1} + \kappa_j \tilde{y}_t - \lambda_j \hat{p}_{j,t}$$

where κ_j and λ_j depend on θ_j

- ▶ Dynamic IS equation:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma} \left[\hat{i}_t - E_t \pi_{t+1} - \hat{r}_t^n \right]$$

Key Equations in the NK Framework

Result: Relative price between sectors becomes state variable. Rich sectoral dynamics.

- ▶ With J parallel currencies, the following $(2J + 2)$ -equation system summarises dynamics in the economy

$$\tilde{y}_t = E_t [\tilde{y}_{t+1}] - \sigma^{-1} \left(\hat{i}_t - \mathbf{v}' E_t [\boldsymbol{\pi}_{t+1}] - r_t^n \right) \quad (1)$$

$$\boldsymbol{\pi}_t = \beta E_t [\boldsymbol{\pi}_{t+1}] + \boldsymbol{\kappa} \tilde{y}_t - \boldsymbol{\lambda} \circ \hat{\boldsymbol{\rho}}_t \quad (2)$$

$$\hat{\boldsymbol{\rho}}_t = \hat{\boldsymbol{\rho}}_{t-1} + (\mathbf{I} - \mathbf{1} \mathbf{v}') (\boldsymbol{\pi}_t + \Delta \mathbf{e}_t) \quad (3)$$

$$\hat{i}_t = \phi_\pi \mathbf{v}' \boldsymbol{\pi}_t + \phi_y \tilde{y}_t \quad (4)$$

where \circ is an operator for element-wise multiplication.

- ▶ Generalised aggregate inflation:

$$\pi_t = \beta E_t [\pi_{t+1}] + \mathbf{v}' \boldsymbol{\kappa} \tilde{y}_t - \mathbf{v}' (\boldsymbol{\lambda} \circ \hat{\boldsymbol{\rho}}_t) + \mathbf{v}' \Delta \mathbf{e}_t$$

Baseline Cases

- ▶ **Proposition 2 (homogeneous rigidity):** Between any two sectors j and j' with homogeneous price rigidity θ ,
 1. the optimal prices in both sectors are equivalent, $p_{j,t}^* + e_{j,t} = p_{j',t}^* + e_{j',t}$;
 2. the bilateral relative price is an autoregressive process,
$$s_{jj',t} = \theta (s_{jj',t-1} + \Delta e_{j,t} - \Delta e_{j',t});$$
 3. the inflation differential is linear in bilateral relative price, $\pi_{j,t} - \pi_{j',t} = -\frac{1-\theta}{\theta} s_{jj',t}$;
 4. the output-gap differential is linear in bilateral relative price, $\tilde{y}_{j,t} - \tilde{y}_{j',t} = -\epsilon s_{jj',t}$.
- ▶ **Proposition 3:** The new Keynesian Philips curve for aggregate inflation is independent of the relative price dynamics if price rigidity is homogeneous across all currency sectors:

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa \tilde{y}_t + \mathbf{v}' \Delta \mathbf{e}_t$$

- ▶ **Proposition 4 (single flexible sector):** An exchange-rate shock to any non-dollar currency j does not spillover to the other currency sectors if prices are perfectly flexible in sector j .

Monetary Policy

What should monetary policy target?

- ▶ Aggregate inflation? Or dollar inflation only?
- ▶ Aggregate output gap? Or dollar sector output gap only?

Thus:

- ▶ Two sector: dollar vs non-dollar
- ▶ Size of non-dollar sector $v = 0.2$
- ▶ Taylor rules

$$\hat{i}_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t \quad (\text{AIAO})$$

$$\hat{i}_t = \phi_\pi \pi_{1,t} + \phi_y \tilde{y}_t \quad (\text{DIAO})$$

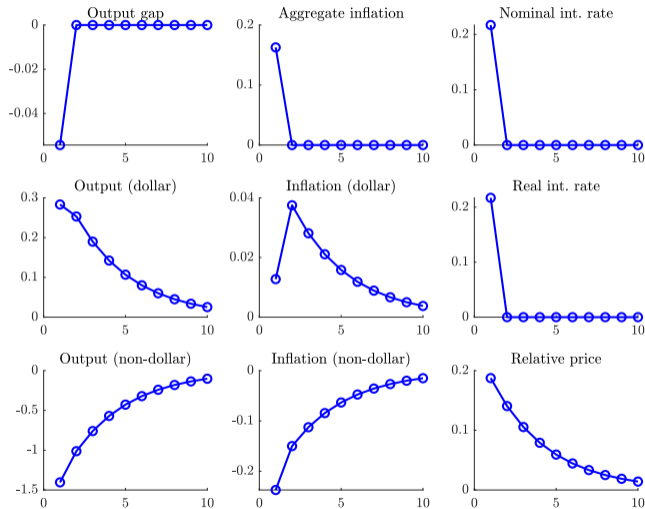
$$\hat{i}_t = \phi_\pi \pi_{1,t} + \phi_y \tilde{y}_{1,t} \quad (\text{DIDO})$$

Parameterization

Table: Parameter values in benchmark model.

Parameter	Value	Description
α	0.250	Share of labour input in production function
σ	1.000	Coefficient of risk aversion
φ	5.000	Inverse Frisch elasticity of labour supply
β	0.990	Discount factor
θ_1	0.750	Probability of not adjusting prices in dollar sector
θ_2	0.750	Probability of not adjusting prices in non-dollar sector
ϵ	9.000	Elasticity of substitution among consumption goods
ϕ_π	1.500	Interest-rate reaction to inflation
ϕ_y	0.125	Interest-rate reaction to output gap
v	0.200	Size of non-dollar sector
$\sigma_{\Delta e}$	0.250	Standard deviation of exchange-rate shock

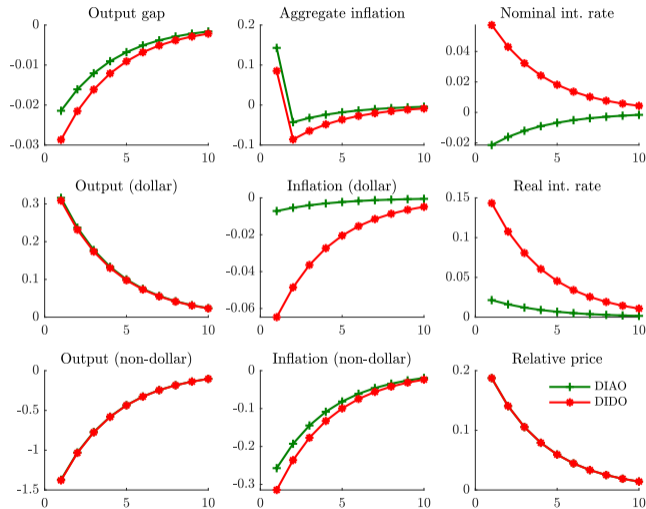
IRFs to dollar depreciation: Baseline Taylor Rule “AIAO”



Result:

Considerable persistent reallocation between sectors.
Large decline in non-dollar sector.
Small and temporary aggregate recession.

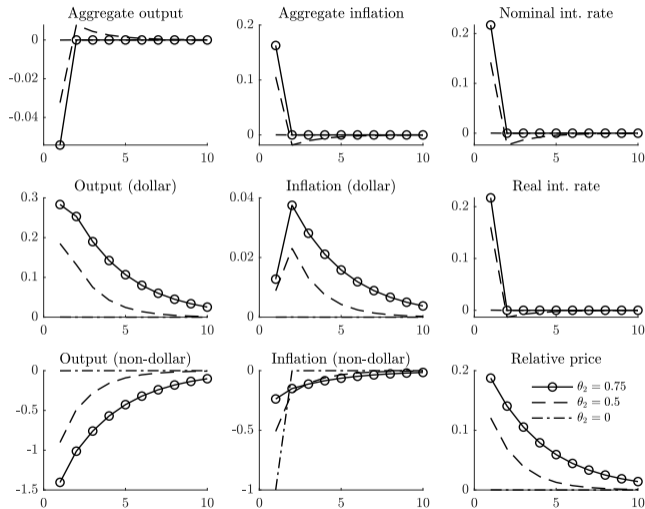
IRFs to dollar depreciation: Alternative Taylor Rules “DIAO” and “DIDO”



Result:

Persistent aggregate recession.

Heterogeneous rigidity



IRFs to dollar depreciation.

- Prices more flexible in non-dollar sector:
 $\theta_1 = 0.75, \theta_2 \in \{0, 0.5, 0.75\}$.

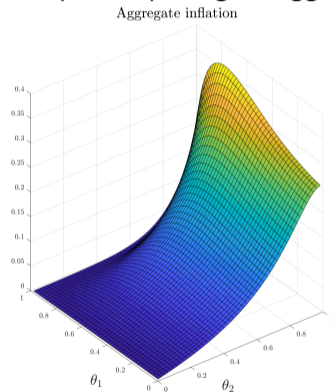
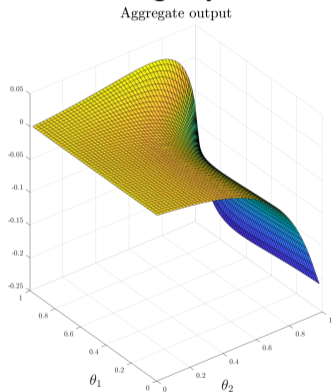
Result:

- Flexibility of prices in non-dollar sector mitigates output drop in that sector and sectoral reallocation. None at flexible limit.
- Subtle: aggregate output persistence.

Volatility

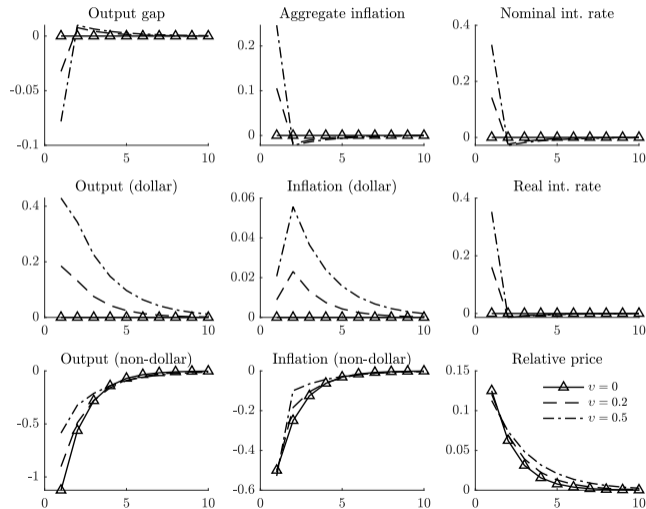
Comparison across parameters. 8-period cummulated IRFs to dollar depreciation.

Result: more non-dollar rigidity induces larger output drops, higher aggregate inflation.



Different sector shares

IRFs to dollar depreciation. $\theta_1 = 0.75$; $\theta_2 = 0.5$.



Result:

Larger non-dollar sector share induces deeper overall recession, higher inflation, larger gain to dollar sector.

Currency Choice

- ▶ Firms choose pricing currencies when given opportunity to re-price
- ▶ We use a discrete choice model. Let $\mathcal{U}_{j,t|t}$ denote the utility of a producer, who resets price in period t .

$$\mathcal{U}_{j,t|t} = \log(\gamma_j \mathcal{V}_{j,t|t}) + \varepsilon_{j,t} \quad (5)$$

where $\mathcal{V}_{j,t|t}$ is the value to the firm upon resetting the price

- ▶ A firm chooses to price in currency j if the utility from the optimised value is at least as high as all the other alternatives:

$$\mathcal{U}_{j,t|t}^* \geq \mathcal{U}_{j',t|t}^* \forall j' = 0, \dots, K. \quad (6)$$

which holds when

$$\varepsilon_{j',t} \leq \log\left(\frac{\gamma_j \mathcal{V}_{j,t|t}^*}{\gamma_{j'} \mathcal{V}_{j',t|t}^*}\right) + \varepsilon_{j,t} \forall j' = 0, \dots, K. \quad (7)$$

Currency Choice

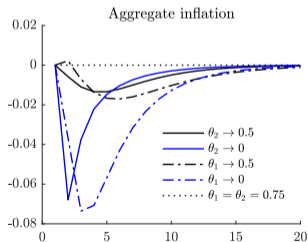
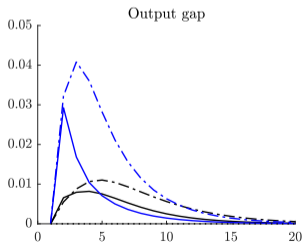
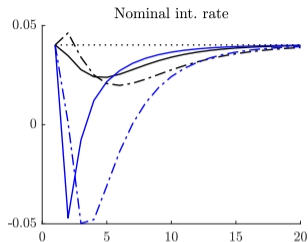
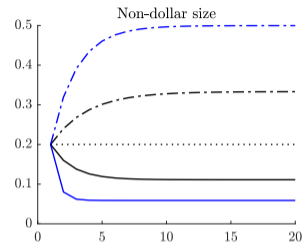
- ▶ The joint probability from K currencies gives the probability of a firm pricing in currency j :

$$\Pr_{j,t} = \frac{\gamma_j \mathcal{V}_{j,t|t}^*}{\sum_{j'=1}^K \gamma_{j'} \mathcal{V}_{j',t|t}^*} \quad (8)$$

- ▶ The law of motion for $v_{j,t}$ is:

$$v_{j,t} = \theta_j v_{j,t-1} + \Pr_{j,t} \sum_{j'=1}^K (1 - \theta_{j'}) v_{j',t-1} \quad (9)$$

Currency choice: transition dynamics



“MIT shock”: dollar sector (“dashed”) or non-dollar sector (“solid”) becomes more flexible.

Result:

Less flexible sector increases.
Slight, persistent boom, inflation drop.

Conclusion

- ▶ Increasing varieties of privately issued digital currencies.
- ▶ **Question:** What happens, when firms price in these currencies, rather than the official currency?
- ▶ **Approach:** an NK model with multiple currencies.
- ▶ **Results:** Relative price between sectors becomes state variable. Rich sectoral dynamics. In response to a dollar depreciation:
 - ▶ Considerable persistent reallocation between sectors. Large decline in non-dollar sector. Small, temporary aggregate recession with AIAO, persist. w. DIAO, DIDO.
 - ▶ Increased flexibility of prices in non-dollar sector mitigates output drop in that sector and sectoral reallocation. None at flexible limit.
 - ▶ Larger non-dollar sector share induces deeper overall recession, higher inflation, larger gain to dollar sector.

Endogenous currency choice: less flexible sector increases substantially. Slight, persistent aggregate boom, inflation drop.