Incorporating Diagnostic Expectations into the New Keynesian Framework

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Introduction

- What are Diagnostic Expectations (DE)?
  - "Representativeness heuristic" (Kahneman & Tversky)
  - Tendency to exaggerate how representative a small sample is
  - Advantages: Microfounded & tractable; realistic & portable

- DE can be productively integrated into the NK framework
  How do we show this?
  First: Start off with technical contribution: solution method

  Then:

  A) Analytically, address 4 key issues
     1. Amplification
     2. Supply shocks
     3. Fiscal policy
     4. Overreaction of expectations

  B) Empirically
     - Show DE improve the fit of medium-scale models

Diagnostic Expectations

- Consider the process

$$x_t = \rho_x x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

- Diagnostic pdf is defined as

$$f_t^\theta(x_{t+1}) = \underbrace{f(x_{t+1}|G_t)}_{true\ \ pdf} \cdot \underbrace{\left[\frac{f(x_{t+1}|G_t)}{f(x_{t+1}|-G_t)}\right]^\theta}_{distortion} \cdot C, \quad \theta > 0$$

- Information sets:
  - G_t: current state t
  - $-G_t$: reference state, here $t-1$.
    (Follow Bordalo, Gennaioli & Shleifer (2018))

θ: degree of diagnosticity

More on Information Sets

Notation: $\check{x}_t$ is the realization of x_t

- G_t: current state t. $\Rightarrow G_t = \{x_t = \check{x}_t\}$
- $-G_t$: reference state, here $t-1$. $\Rightarrow -G_t = \{x_t = \rho_x \check{x}_{t-1}\}$
- The "two minds" of the diagnostic agent
  One mind is inattentive

Formula for Univariate Case and Example

- Diagnostic expectation is:

$$\mathbb{E}_t^{\theta}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta(\mathbb{E}_t[x_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}])$$

(Bordalo, Gennaioli & Shleifer (2018), henceforth BGS)

- We have that:

$$\mathbb{E}_t[x_{t+1}] = \rho_x \check{x}_t \text{ and } \mathbb{E}_{t-1}[x_{t+1}] = \rho_x^2 \check{x}_{t-1}$$

- So:

$$\mathbb{E}_t^{\theta}[x_{t+1}] = \rho_x \check{x}_t + \theta(\rho_x \check{x}_t - \rho_x^2 \check{x}_{t-1}) = \rho_x \check{x}_t + \theta \rho_x \check{\varepsilon}_t$$

$\Longrightarrow$ extrapolation

General Model

- Exogenous process

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{v}_t$$

- Recursive model:

$$\mathbb{E}_t^{\theta}[\mathbf{F}\mathbf{y}_{t+1} + \mathbf{G_1}\mathbf{y}_t + \mathbf{M}\mathbf{x}_{t+1} + \mathbf{N_1}\mathbf{x}_t] + \mathbf{G_2}\mathbf{y}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{N_2}\mathbf{x}_t = 0$$

- **Question:** How to compute the equilibrium $\mathbb{E}_t^{\theta}[\mathbf{F}\mathbf{y}_{t+1} + \ldots]$?
  1. Equilibrium $\mathbf{y}_t$?
  2. Combinations of future and contemporaneous vars?

Combinations of Future and Contemporaneous Variables

Predetermined variables cannot be treated as constants

- Consider: $y_t = \rho_y y_{t-1} + \eta_t$ and $\mathbb{E}_t^\theta[x_{t+1} + y_t]$

  REMARK: $\mathbb{E}_t^\theta[x_{t+1} + y_t] \neq \mathbb{E}_t^\theta[x_{t+1}] + y_t$

  Intuition: DE generate "behavioral inattentiveness"
  One mind of the agent is inattentive
  $(-G_t = \{x_t = \rho_x \check{x}_{t-1}; y_t = \rho_y \check{y}_{t-1}\}$ and so $\check{\eta}_t \notin -G_t)$

- Can nevertheless reestablish a strong form of additivity
  - This result delivers RE Representation
  - **But notice:** model needs to be loglinearized from scratch

Example: Loglinear Approximation of Euler Equation

- Consider

$$\frac{u'(C_t)}{P_t} = \beta(1+i_t)\mathbb{E}_t^{\theta}\left[\frac{u'(C_{t+1})}{P_{t+1}}\right]$$

- Notice!

$$\mathbb{E}_t^{\theta}[X_{t+1}Y_t] \neq \mathbb{E}_t^{\theta}[X_{t+1}]Y_t$$

- Hence, use conditioning on $t-1$:

$$u'(C_t)\frac{P_{t-1}}{P_t} = \beta(1+i_t)\mathbb{E}_t^{\theta}\left[u'(C_{t+1})\frac{P_{t-1}}{P_t}\frac{P_t}{P_{t+1}}\right]$$

and approximate

Obtaining Log-Linear Approximation

- We have:

$$u'(C_t)\frac{P_{t-1}}{P_t} = \beta(1+i_t)\mathbb{E}_t^{\theta}\left[u'(C_{t+1})\frac{P_{t-1}}{P_t}\frac{P_t}{P_{t+1}}\right]$$

- Resulting diagnostic Fisher equation:

$$\hat{r}_t = \hat{i}_t - \mathbb{E}_t[\pi_{t+1}] \underbrace{-\theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])}_{P_{t-1}/P_t} \underbrace{-\theta(\mathbb{E}_t[\pi_{t+1}] - \mathbb{E}_{t-1}[\pi_{t+1}])}_{P_t/P_{t+1}}$$

- Appendix presents loglinearization steps of full DSGE

Implications for New Keynesian Model

- Model

$$\hat{y}_t = \mathbb{E}_t^{\theta}[\hat{y}_{t+1}] - (\hat{i}_t - \mathbb{E}_t^{\theta}[\pi_{t+1}]) + \theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])$$
$$\pi_t = \beta \mathbb{E}_t^{\theta}[\pi_{t+1}] + \kappa(\hat{y}_t - \hat{a}_t)$$
$$\hat{i}_t = \phi_\pi \pi_t + \phi_x(\hat{y}_t - \hat{a}_t)$$

- Euler equation combines both DE and RE
- $\theta = 1$ (Bordalo et al. 2020)

Amplification: NK vs. RBC

- New Keynesian Model

Variable	RE	DE	Percentage Increase
Output	0.0048	0.0085	77%

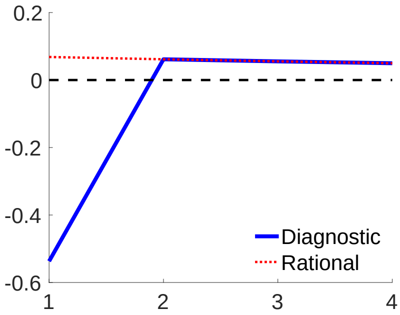
Volatility of output **increases**

- (Frictionless) Real Business Cycle Model

Variable	RE	DE	Percentage Increase
Output	0.0064	0.0059	-7%
Consumption	0.0015	0.0030	100%
Investment	0.0533	0.0503	-6%

Volatility of output **falls**

"Covid" Shock:
Fall of Output Gap After Negative TFP Shock



Intuition: DE agent expects TFP to fall by a lot
(in excess of reality)
$\Longrightarrow$ Sharp drop in consumption

Fiscal Policy

Proposition

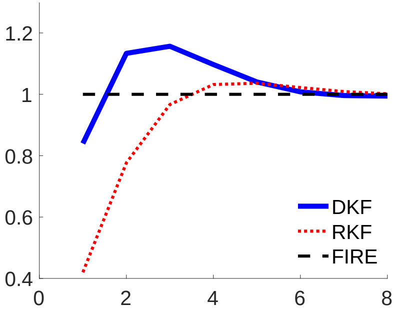
Consider i.i.d. government spending shocks.

1. *Under DE, the multiplier is greater than 1 iif* $\theta > \phi_\pi$.
2. *The multiplier is greater under DE than under RE.*
3. *The multiplier is increasing in* θ, *and tends to* ∞ *as* $\theta \to \phi_\pi + \kappa^{-1}$.

- Diagnostic Fisher equation:
  $\hat{r}_t = \hat{i}_t - \mathbb{E}_t^\theta[\pi_{t+1}] - \theta(\pi_t - \mathbb{E}_{t-1}[\pi_t])$
- Role of **endogenous** extrapolation of inflation
- Dominates effect of monetary policy if $\theta > \phi_\pi$

Introducing Imperfect Info: Diagnostic Kalman Filter

Investigate in Blanchard, L'Huillier & Lorenzoni (2013).



Short-run underreaction, delayed overreaction, and humps.

Bayesian Estimation

- Rich model with host of frictions and shocks

  Question: Do DE improve the fit to the data, even in the presence of all these other nominal, real, and informational frictions?

- θ post. mode: 0.99, conf. interval: [0.77; 1.21]

  Marginal likelihoods:
  - RE model: -1590.66
  - DE model: improvement to -1584.31

- $2\log(BF) = 12.70$
  Strong evidence in favor of DE

Summary

- How to integrate diagnostic expectations into linear models
- Rich insights in the context of NK models
- Better fit to business cycle data