

A NOMINAL DEMAND AUGMENTED PHILLIPS CURVE

Marcus Hagedorn

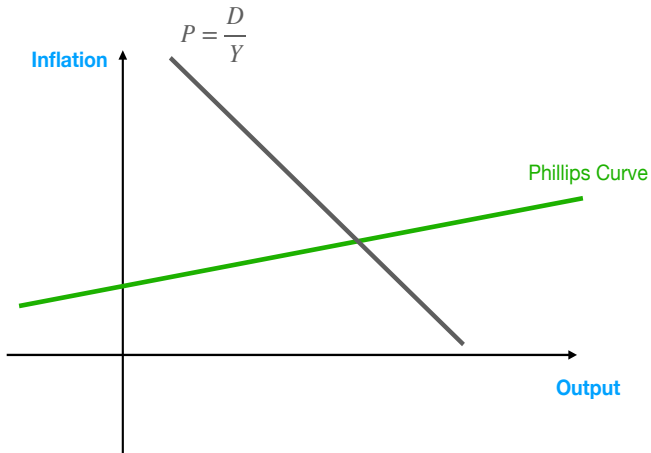
University of Oslo and CEPR

Bank of Finland and CEPR Joint Conference

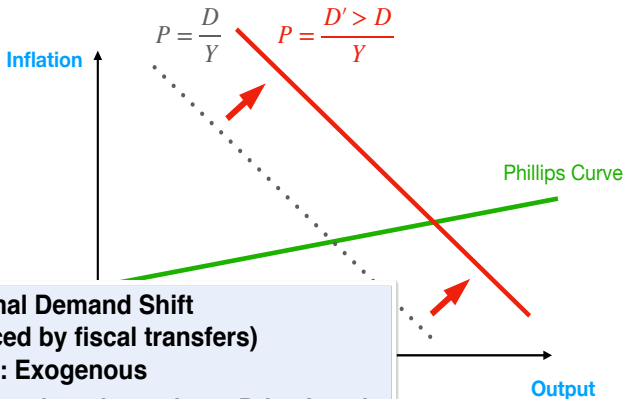
Helsinki, 12-13 September 2024

NEW KEYNESIAN PHILLIPS CURVE

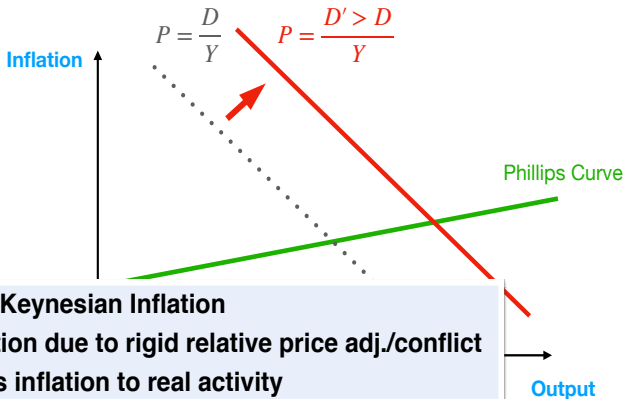
ONE PERIOD VERSION



DEMAND CURVE SHIFT

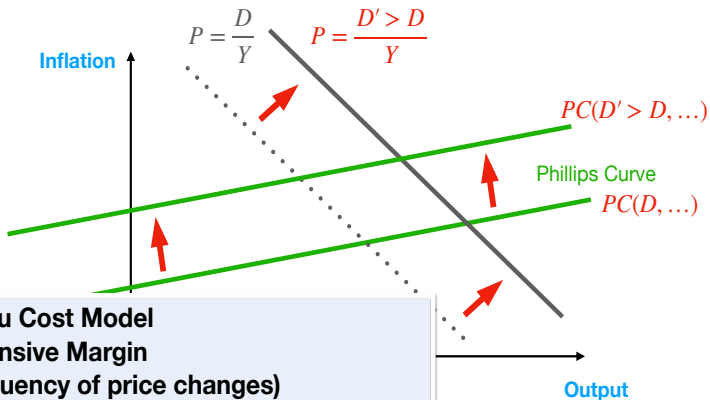


DEMAND CURVE SHIFT



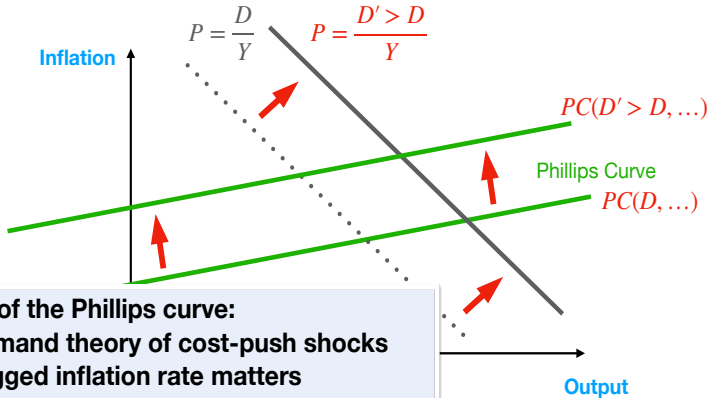
- New Keynesian Inflation
- Inflation due to rigid relative price adj./conflict
- Links inflation to real activity
- Intensive Margin (size of price changes)

PHILLIPS CURVE SHIFT



- Menu Cost Model
- Extensive Margin (frequency of price changes)
- Nominal demand
 - > Inflation and real activity decoupled

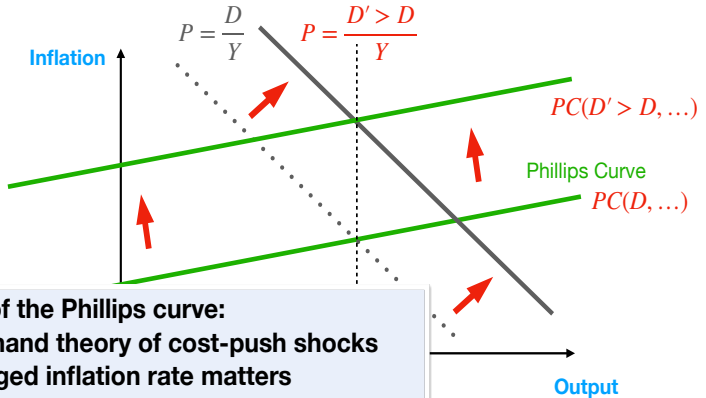
PHILLIPS CURVE SHIFT



Shift of the Phillips curve:

- Demand theory of cost-push shocks
- Lagged inflation rate matters (captures demand shock history)
- History matters -> persistent inflation

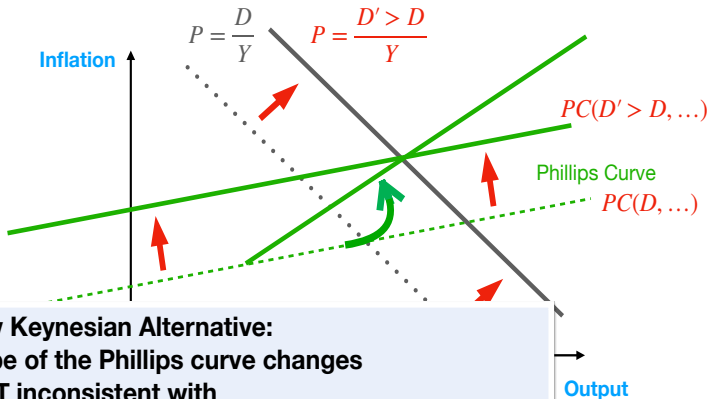
PHILLIPS CURVE SHIFT



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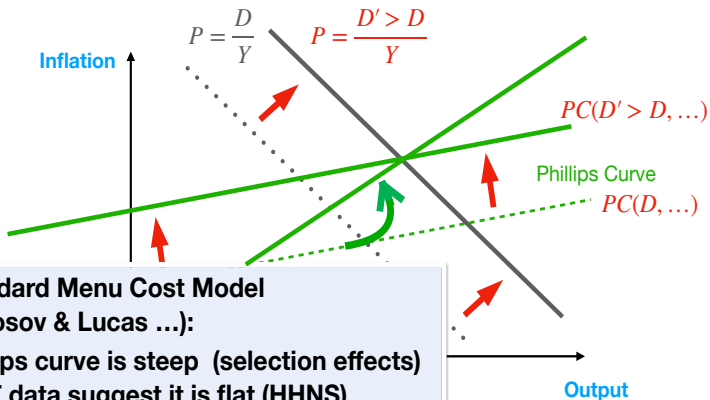
PHILLIPS CURVE SHIFT



New Keynesian Alternative:
Slope of the Phillips curve changes
[BUT inconsistent with

- Hazell, Herreno, Nakamura and Steinsson (2022)
- Price change frequency]

PHILLIPS CURVE SHIFT



Standard Menu Cost Model

(Golosov & Lucas ...):

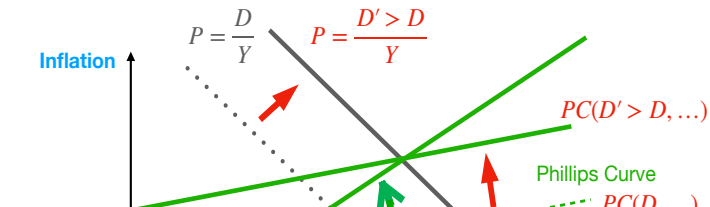
Phillips curve is steep (selection effects)

[BUT data suggest it is flat (HHNS)

(missing disinflation/reinflation)]

AND slope not changing much

PHILLIPS CURVE SHIFT



Standard Menu Cost
(Golosov & Lucas ...)
Phillips curve is steep
[BUT data suggest it
(missing disinflation/rennation)]
AND slope not changing much

Standard Menu Cost Model

- $D = \text{Nominal GDP} = \text{Money } M$
- Monetary policy sets M :
-> Controls Nom. GDP D
- More General:
 $D(\text{Nom. Rate, Fiscal Policy, Shocks, ...})$

NOMINAL DEMAND AUGMENTED PHILLIPS CURVE

Summarizing:

Menu-cost model:	Intensive Margin	& Extensive Margin
Phillips curve:	Moving along the PC	& Shift of the PC
Integrating:	New Keynesian	& “Helicopter” Inflation

NOMINAL DEMAND AUGMENTED PHILLIPS CURVE

Summarizing:

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Today:

Empirical Evidence [building on Hazell et. al.] and Theory

1. Nominal Demand Augmented Phillips Curve
 - ▶ New Keynesian real activity component
 - ▶ Nominal demand component
2. Lagged Inflation

STATE-DEPENDENT PRICE SETTING

- ▶ Firms indexed by $i \in [0, 1]$ set price $p_t(i)$.
- ▶ Idiosyncratic price **adjustment costs**.
- ▶ Nominal demand D_t and price level $P_t = \left[\int_0^1 p_t(i)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$
- ▶ **Prob. of adjusting** its price $p_{t-1}(i)$ prior to knowing adj. cost:

$$\Lambda_t(p_{t-1}(i), P_t, D_t)$$

- ▶ **Aggregate price dynamics**:

$$= \frac{\Pi_t^{1-\epsilon} - 1 \int_0^1 [p_t^*(i)^{1-\epsilon} - p_{t-1}(i)^{1-\epsilon}] \Lambda(p_{t-1}(i), P_t, D_t) di}{P_{t-1}^{1-\epsilon}}$$

PRICE SETTING MODEL

NOMINAL DEMAND AUGMENTED PHILLIPS CURVE (NDPC)

- **Log-linearizing** [Caballero & Engel 2007]

$$\pi_t = (\hat{p}_t^* - \hat{P}_{t-1}) \underbrace{\int_0^1 \Lambda(p_{t-1}(i), P_{t-1}, D_{t-1}) di}_{=:\bar{\Lambda}} \quad \text{[Intensive Margin]}$$
$$+ \underbrace{\int_0^1 \frac{1 - \left(\frac{p_{t-1}(i)}{P_{t-1}}\right)^{1-\epsilon}}{1-\epsilon} \hat{\Lambda}(i) di}_{=:\phi} \quad \text{[Extensive Margin]}$$

- **Linearizing extensive margin** ($\phi^D \geq 0$, $\phi^P \leq 0$)

$$\phi = \phi^D \Delta D_t + \phi^P \pi_t.$$

- The **optimal price** [One Period model]:

$$\hat{p}_t^* - \hat{P}_{t-1} = \hat{m}c_t + \hat{P}_t - \hat{P}_{t-1},$$

- **Solving** for the **inflation rate** π_t :

$$\pi_t = \underbrace{\zeta}_{\geq 0} \hat{m}c_t + \underbrace{\Phi^D}_{\geq 0} \Delta D_t.$$

PRICE SETTING MODEL

NDPC - INFINITE HORIZON

- **Optimal Price** [Dotsey, King & Wolman 1999]

$$(\hat{p}_t^* - \hat{P}_{t-1}) = \Xi E_t \sum_{k=0}^{\infty} \beta^k \lambda_{k,t} [\hat{m}c_{t+k} + (\hat{P}_{t+k} - \hat{P}_{t-1})]$$

- Using this in $\pi_t = (\hat{p}_t^* - \hat{P}_{t-1})\bar{\Lambda} + \phi_t$ and rearranging:

$$\pi_t = \frac{(\bar{\Lambda} \Xi)}{1 - \bar{\Lambda}} E_t \sum_{k=0}^{\infty} \beta^k \hat{m}c_{t+k} + \frac{\Phi_t}{1 - \bar{\Lambda}}$$

- Φ_t depends on $\Delta D_t, \Delta D_{t-1}, \Delta D_{t-2}, \dots$

$$\pi_t = \zeta E_t \sum_{k=0}^{\infty} \beta^k \hat{m}c_{t+k} + \Phi_D^0 \Delta D_t + \sum_{k=1}^{\infty} \Phi_D^k \Delta D_{t-k}$$

REGIONAL NDPC

Regional NDPC for Non-Tradables:

$$\pi_{r,t}^N = E_t \sum_{k=0}^{\infty} \beta^k (\zeta \hat{m}c_{r,t+k}^N) + \Phi_D^0 \Delta D_{r,t}^N + \sum_{k=1}^{\infty} \Phi_D^k \Delta D_{r,t-k}^N$$

- ▶ $\pi_{r,t}^N$ - non-tradable inflation in region r .
- ▶ D_r^N - nominal demand for non-tradables in region r .
- ▶ $\hat{m}c_{r,t+k}^N = -\varphi^{-1} \hat{u}_{r,t} - \hat{p}_{r,t}^N$ [$mc^N = w/P^N$; $u \approx -w/P$]
 - ▶ $\hat{u}_{r,t}$ - unemployment rate in region r
 - ▶ $\hat{p}_{r,t}^N$ - relative price of non-tradables

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- ▶ **Incomplete markets:**
Nominal transfers \rightarrow Nominal Demand.

REGIONAL NDPC

Regional NDPC for Non-Tradables:

$$\pi_{r,t}^N = E_t \sum_{k=0}^{\infty} \beta^k (\zeta \hat{m}c_{r,t+k}^N) + \Phi_T^0 \Delta T_{r,t} + \sum_{k=1}^{\infty} \Phi_T^k \Delta T_{r,t-k}$$

- ▶ $\pi_{r,t}^N$ - non-tradable inflation in region r .
- ▶ D_r^N - nominal demand for non-tradables in region r .
- ▶ $\hat{m}c_{r,t+k}^N = -\varphi^{-1} \hat{u}_{r,t} - \hat{p}_{r,t}^N$ [$mc^N = w/P^N$; $u \approx -w/P$]
 - ▶ $\hat{u}_{r,t}$ - unemployment rate in region r
 - ▶ $\hat{p}_{r,t}^N$ - relative price of non-tradables
- ▶ **Incomplete markets:**
 - Nominal transfers \rightarrow Nominal Demand.
- ▶ T_r - nominal transfer to region r households.

DATA

▶ Regional Transfers

- ▶ BEA- Personal Current Transfer Receipts (SQINC35)
- ▶ “State unemployment insurance compensation”
+ “All other personal current transfer receipts”
- ▶ Includes income maintenance benefits (SSI, EITC, SNAP), Alaska Permanent Fund benefits and ARRA transfers

▶ State-Level Inflation Rates and Price Indices

- ▶ Data from Hazell, Herreño, Nakamura and Steinsson (2022)
- ▶ Use CPI Research Database provided by BLS
- ▶ Non-shelter state inflation rates
- ▶ Non-tradables: education, telephone, medical and recreational services, ...

▶ Labor market data

- ▶ State-level unemployment rates from LAUS.
- ▶ State-industry level employment data from QCEW.

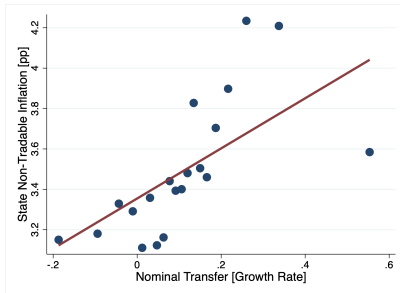
VISUALISATION

BINNED SCATTERPLOTS

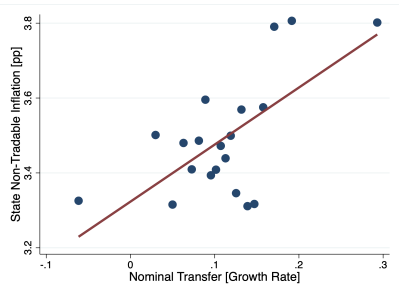
State Nominal transfer growth rate



State Non-Tradable Inflation



No State and Time Fixed Effects



State and Time Fixed Effects

EMPIRICAL SPECIFICATION

$$\pi_{r,t}^N = \kappa E_t \sum_{k=0}^K \beta^k \hat{m} c_{r,t+k}^N + \Phi \Delta T_{r,t-4} + \gamma \pi_{r,t-4}^N + \alpha_r + \gamma_t + \epsilon_{r,t}$$

- ▶ $\pi_{r,t}^N = \log(P_{r,t}^N) - \log(P_{r,t-4}^N)$ [Non-tradable inflation]
- ▶ $\Delta T_{r,t-4} = \log(T_{r,t-4}) - \log(T_{r,t-8})$ [State Transfer]
- ▶ $\sum_{k=0}^K \beta^k \hat{m} c_{r,t+k}^N$ truncated at $K = 20$ quarters
- ▶ $\pi_{r,t-4}^N$ - Lagged non-tradable inflation rate
- ▶ α_r, γ_t : State and time fixed effects.

IDENTIFICATION

BARTIK-TYPE INSTRUMENT

Tradable Bartik type instrument in region r in period t :

$$\mathcal{B}_{r,t} := \sum_l \bar{z}_{l,r} \times g_{-r,l,t},$$

- ▶ $\bar{z}_{l,r}$ is the time-average employment share of industry l in the tradable sector in region r
- ▶ $g_{-r,l,t}$ is the Period t three-year growth of industry r national employment leaving out region r

Idea:

- ▶ $g_{-r,l,t} \rightarrow$ differential impact on NT-demand in Sector r depending on $\bar{z}_{l,r}$
- ▶ Example [HHNS]: Oil price shock affects Texas and Illinois differently
- ▶ No time correlation with the Texas/Illinois cost-difference.

IDENTIFICATION

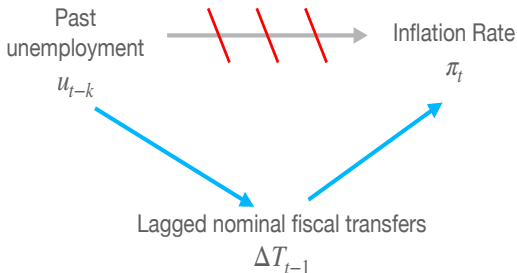
PAST UNEMPLOYMENT RATES

Relevance:

$u_{r,t-k}$ triggers UI and other income maintenance programs:

Orthogonality:

$$E(u_{r,t-k}\epsilon_{r,t}) = 0, \quad k = \underline{k}, \underline{k} + 1, \dots, \bar{k}$$



IDENTIFICATION

PAST UNEMPLOYMENT RATES

Orthogonality:

$$E(u_{r,t-k}\epsilon_{r,t}) = 0, \quad k = \underline{k}, \underline{k} + 1, \dots, \bar{k}$$

Is correct

- ▶ in New Keynesian model if $\hat{m}c_t = u_t$:

$$\pi_{r,t}^N = \kappa \sum_{k=0}^{\infty} \beta^k \hat{m}c_{r,t+k}^N$$

- ▶ in menu cost model if $\hat{m}c_t = u_t$ and control for ΔD_{t-k}

⇒ I impose this [theoretical identifying assumption](#)

$$[\hat{m}c_t = u_t] + [\text{control } \Delta D_{t-k}] \rightarrow E(u_{r,t-k}\epsilon_{r,t}) = 0$$

IDENTIFICATION

PAST UNEMPLOYMENT RATES

Orthogonality:

$$E(u_{r,t-k}\epsilon_{r,t}) = 0, \quad k = \underline{k}, \underline{k} + 1, \dots, \bar{k}$$

BUT:

- ▶ Unemployment \neq Marginal cost gap \Rightarrow Residual

$$\hat{m}c_{r,t}^N = u_{r,t} + mc_{r,t}^{res}, \quad E(u_{r,t}mc_{r,t}^{res}) = 0.$$

- ▶ Inflation:

$$\pi_{r,t}^N = \sum_{k=0}^{\infty} \beta^k u_{r,t+k} + \underbrace{\sum_{k=0}^{\infty} \beta^k mc_{r,t+k}^{res}}_{\hookrightarrow \epsilon_{r,t}}$$

- ▶ Orthogonality condition then reads:

$$E(u_{r,t-k}mc_{r,t}^{res}) = 0, \quad k = \underline{k}, \underline{k} + 1, \dots, \bar{k}.$$

- ▶ Overidentifying assumptions are tested.

RESULTS

	ΔT	$\sum mc$	π_{t-4}
Benchmark	0.115 (0.037)	0.049 (0.020)	0.081 (0.111)

Specification Tests

Identification	Under-I.	Weak-I. (F-Test)	Over-I.
Benchmark	0.019 ✓	47.851 ✓	0.287 ✓

Clustered P-values in parentheses.

RESULTS

	ΔT	$\sum mc$	π_{t-4}	ΔT_{t-4}
Benchmark	0.115 (0.037)	0.049 (0.020)	0.081 (0.111)	
Lagged Transfer	0.145 (0.017)	0.067 (0.015)	0.058 (0.268)	0.115 (0.036)

Specification Tests

Identification	Under-I.	Weak-I. (F-Test)	Over-I.
Benchmark	0.019 ✓	47.851 ✓	0.287 ✓
Lagged Transfer	0.007 ✓	21.801 ✓	0.912 ✓✓

Clustered P-values in parentheses.

RESULTS

	ΔT	$\sum mc$	π_{t-4}
Benchmark	0.115 (0.037)	0.049 (0.020)	0.081 (0.111)
No Infl. Lag	0.130 (0.042)	0.052 (0.025)	
<i>Specification Tests</i>			
Identification	Under-I.	Weak-I. (F-Test)	Over-I.
Benchmark	0.019 ✓	47.851 ✓	0.287 ✓
No Infl. Lag	0.028 ✓	46.776 ✓	0.282 ✓

Clustered P-values in parentheses.

RESULTS

	ΔT	$\sum mc$	π_{t-4}
Benchmark	0.115 (0.037)	0.049 (0.020)	0.081 (0.111)
No Marg. Cost	0.052 (0.012)		0.086 (0.106)

Specification Tests

Identification	Under-I.	Weak-I. (F-Test)	Over-I.
Benchmark	0.019 ✓	47.851 ✓	0.287 ✓
No Marg. Cost	0.011 ✓	87.818 ✓	0.211 ✓

Clustered P-values in parentheses.

RESULTS

	ΔT	$\sum mc$	π_{t-4}
Benchmark	0.115 (0.037)	0.049 (0.020)	0.081 (0.111)
No Transfer		0.021 (0.171)	0.080 (0.107)

Specification Tests

Identification	Under-I.	Weak-I. (F-Test)	Over-I.
Benchmark	0.019 ✓	47.851 ✓	0.287 ✓
No Transfer	0.001 ✓	156.5 ✓	0.030 ✗

Clustered P-values in parentheses.

RESULTS

DIFFERENT MARGINAL COST MEASURES

	Marg. Cost $\beta = 0.95$	Marg. Cost $\beta = 0.9$	Marg. Cost $\varphi = 1/2$	Marg. Cost $\varphi = 2$
ΔT	0.108 (0.023)	0.098 (0.013)	0.115 (0.037)	0.115 (0.037)
$\sum mc$	-0.060 (0.010)	-0.074 (0.005)	-0.024 (0.020)	-0.097 (0.020)
<i>Specification Tests</i>				
Under-I.	0.009 ✓	0.006 ✓	0.019 ✓	0.019 ✓
Weak-I.	57.070 ✓	66.903 ✓	47.834 ✓	47.877 ✓
Over-I.	0.309 ✓	0.317 ✓	0.287 ✓	0.288 ✓

RESULTS

REAL DEMAND PHILLIPS CURVE ESTIMATES

	Inflation	Inflation
Real Transfer Growth	0.164 (0.938)	
Real Transfer Level		-39.075 (0.454)
Σ Marginal Cost	-0.023 (0.292)	0.180 (0.508)

- ▶ **Nominal** and **not real** demand growth **matters**.
- ▶ Not proxy for **real marginal cost**: Real Level does not matter

RESULTS (AGGREGATE U.S. DATA)

INDEXATION

Looking under the hood of the estimated time FE:

	Estimated Time FE $\hat{\gamma}_t$	
Lag. Inflation, $\pi_{US,t-4}$	0.658 (0.000)	0.103 (0.531)
Lag. Pers. Income gr.		28.438 (0.017)
Lag. Social Security gr.		50.457 (0.004)

RESULTS (AGGREGATE U.S. DATA)

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Year $t - 1$

| Year t

π_{t-1} $\xrightarrow{\text{Indexation, ...}}$ Personal Income growth $_{t-1}$ $\xrightarrow{\text{NDPC}}$ π_t

RESULTS (AGGREGATE U.S. DATA)

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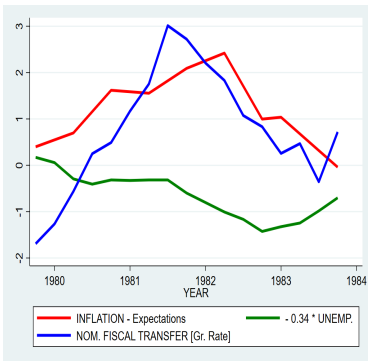
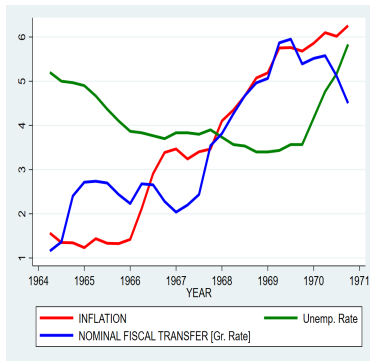
“AGGREGATE IMPLICATIONS”

Predicted inflation using U.S. Aggregate Personal Transfers

$$\pi_t^{pred} = const + 0.145\Delta T_{t-4}^{U.S.} + 0.115\Delta T_{t-8}^{U.S.}$$

U.S. 1964-1970

U.S. 1980 - 1984



CONCLUSION

Theoretical Contribution:

- ▶ Derived nominal demand augmented Phillips curve (**NDPC**)
- ▶ **Two determinants of inflation:**
Real marginal costs and nominal demand growth
- ▶ NDPC \neq NKPC (without strong assumptions)

Empirical Contribution:

- ▶ **Estimate NDPC** in cross-section of U.S. states
- ▶ **Confirm theoretical predictions:** Nominal not real demand growth matters; not a proxy for marg. cost, looks like cost-push shock from NKPC perspective

More Slides

High level modeling choices

- ▶ Follow **Midrigan** (ECMA 2011)
- ▶ Introduce **idiosyncratic firm productivity shocks**
- ▶ Stochastic (exponential) adjustment costs
- ▶ No mass points
- ▶ **Weekly** and quarterly versions

Match key steady state targets:

- ▶ **Frequency** of (regular) weekly price changes: 2.9%.
- ▶ **Distribution** of size of (regular) price changes
- ▶ Replicates evidence on **intensive and extensive margin** in Alvarez, Beraja, Gonzalez-Rozada and Neumeyer (QJE 19):
“From hyperinflation to stable prices: Argentina’s evidence on menu cost models”

EXPERIMENTS

- ▶ Study response of model to shocks to nominal demand growth ΔD_t
- ▶ Linearize model with small MIT-shocks in sequence space (Boppart, Krusell & Mitman 2018, Auclert et al 2021)
- ▶ Consider two processes for demand growth:
 - ▶ Quarterly autocorrelation $\rho_D = 0.5$ (as in the data)
 - ▶ Quarterly autocorrelation $\rho_D = 0$ (\equiv permanent level shock)
- ▶ Simulate to obtain weekly model generated data
- ▶ Implement quarterly Phillips curve regressions:

$$\pi_t = \alpha \sum \mathbb{E}[\beta^k mc_{t+k}] + \gamma \pi_{t-1} + \delta \Delta D_{t-1} + \epsilon_t$$

RESULTS: $\rho_D = 0.5$

New Keynesian Calvo Specifications:

	$\sum mc$	π_{t-1}
Calvo	0.0150 (0.0001)	
+ Lagged Inflation	0.0116 (0.0001)	0.4118 (0.0052)

Standard errors in parentheses.

RESULTS: $\rho_D = 0.5$

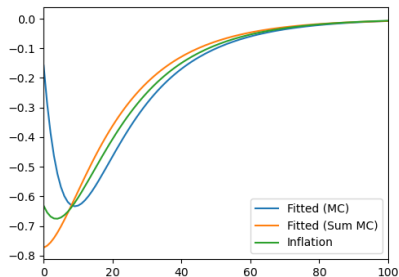
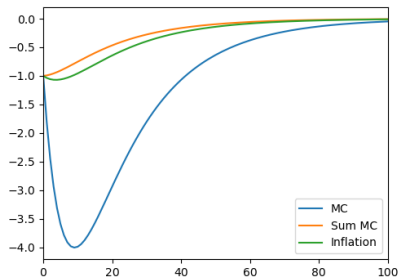
Full specification:

	$\sum mc$	π_{t-1}	ΔD
Calvo	0.0116 (0.0001)	0.4118 (0.0043)	
Full Specification	0.0080 (0.0000)	-0.0271 (0.0036)	0.5862 (0.0039)

Standard errors in parentheses.

UNDERSTANDING THE RESULTS: WEEKLY IRFs $\rho_D = 0.5$

(Expected discounted) Output gaps don't coincide with inflation

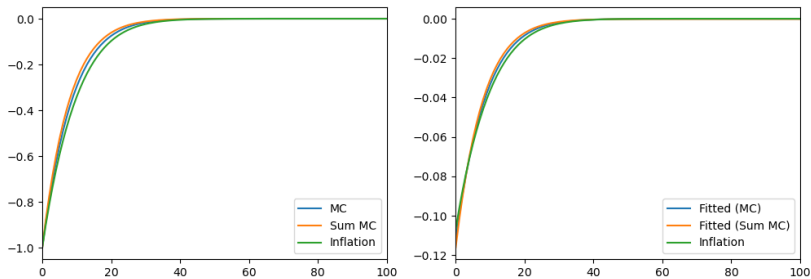


Fitted (MC): Coefficient $\pi_{t-1} = 0.7828$

Fitted (Sum MC): Coefficient $\pi_{t-1} = 0.2907$

UNDERSTANDING THE RESULTS: WEEKLY IRFs $\rho_D = 0$

Nature of shock matters. With level shocks, (expected discounted) output gaps much closer to inflation

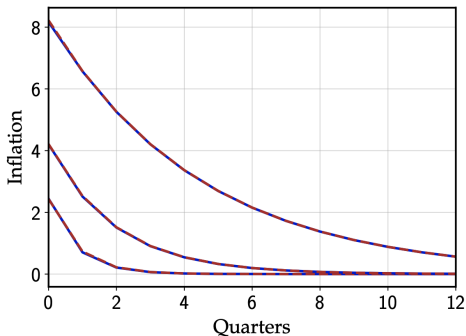


Fitted (MC): Coefficient $\pi_{t-1} = 0.1217$

Fitted (Sum MC): Coefficient $\pi_{t-1} = 0.065$

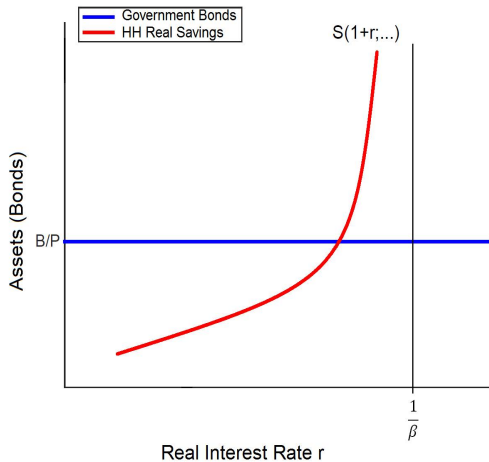
COMPARISON TO AUCLERT ET AL 2023

With AR(1) shocks ($\rho = \{0.3, 0.6, 0.8\}$) to **real marginal costs**, inflation and (expected discounted) output gaps coincide



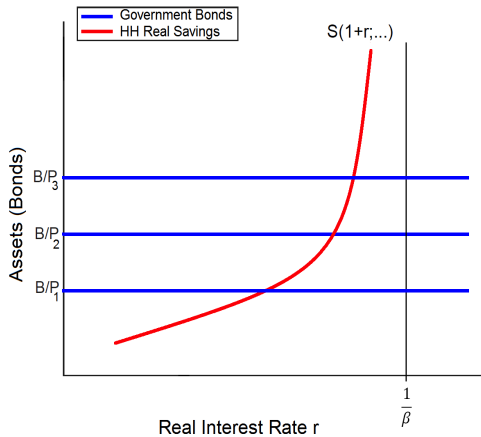
STEADY STATE PRICE LEVEL

ASSET MARKET IN INCOMPLETE MARKETS MODEL

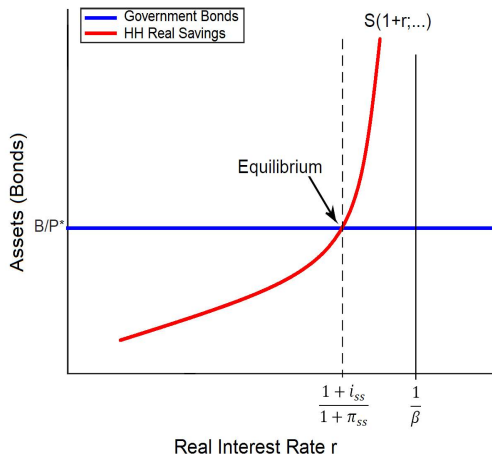


STEADY STATE PRICE LEVEL

INDETERMINACY



STEADY STATE PRICE LEVEL



Real Interest Rate:

$$(1 + r) = \frac{1+i}{1+\pi}$$

Monetary Policy:

Sets $1 + i$

Fiscal Policy:

$$\pi = \frac{B' - B}{B} = \frac{G' - G}{G} = \frac{T' - T}{T}$$

i : nominal interest rate

r : real interest rate

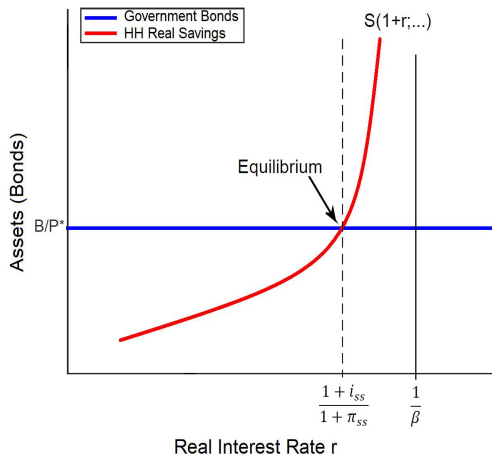
π : inflation rate

B : nominal bonds

G : nominal government spending

T : nominal tax revenue

STEADY STATE PRICE LEVEL



Steady-state Conditions:

$$\frac{B}{P^*} = S\left(\frac{1+i_{ss}}{1+\pi_{ss}}\right)$$

$$1 + \pi_{ss} = 1 + \frac{B' - B}{B}$$

i : nominal interest rate

r : real interest rate

π : inflation rate

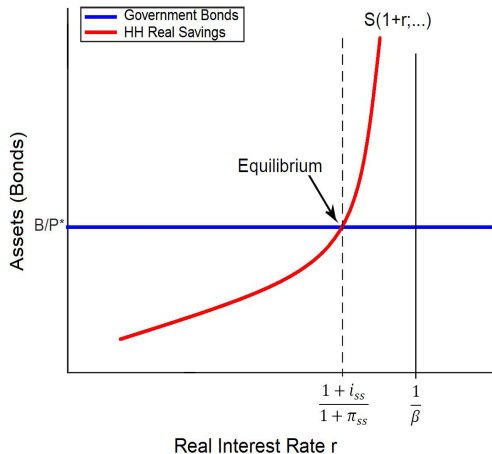
B : nominal bonds

G : nominal government spending

T : nominal tax revenue

STEADY STATE PRICE LEVEL

WITH CAPITAL K



Steady-state Conditions:

$$K + \frac{B}{P^*} = S\left(\frac{1 + i_{ss}}{1 + \pi_{ss}}\right)$$

$$1 + \pi_{ss} = 1 + \frac{B' - B}{B}$$

i : nominal interest rate
 r : real interest rate
 π : inflation rate

B : nominal bonds
 G : nominal government spending
 T : nominal tax revenue

STEADY STATE PRICE LEVEL

WITH ENDOGENOUS MONEY M

Steady-state Conditions

$$\frac{B}{P^*} = S\left(\frac{1 + i_{ss}}{1 + \pi_{ss}}, 1 + \pi_{ss}\right)$$

$$\frac{M}{P^*} = L\left(\frac{1 + i_{ss}}{1 + \pi_{ss}}, 1 + \pi_{ss}\right)$$

$$1 + \pi_{ss} = 1 + \frac{B' - B}{B} = 1 + \frac{M' - M}{M}$$

Central bank provides

$$M = P^* L\left(\frac{1 + i_{ss}}{1 + \pi_{ss}}, 1 + \pi_{ss}\right)$$

STEADY STATE PRICE LEVEL

WITH ENDOGENOUS MONEY M

Steady-state Conditions [Open Market operations]

$$\frac{B - M}{P^*} = S\left(\frac{1 + i_{ss}}{1 + \pi_{ss}}, 1 + \pi_{ss}\right)$$

$$\frac{M}{P^*} = L\left(\frac{1 + i_{ss}}{1 + \pi_{ss}}, 1 + \pi_{ss}\right)$$

$$1 + \pi_{ss} = 1 + \frac{B' - B}{B} = 1 + \frac{M' - M}{M}$$

Central bank provides

$$M = P^* L\left(\frac{1 + i_{ss}}{1 + \pi_{ss}}, 1 + \pi_{ss}\right)$$

STEADY STATE PRICE LEVEL

WITH ENDOGENOUS MONEY M

Steady-state Conditions [Open Market operations]

$$\frac{B - M}{P^*} = S\left(\frac{1 + i_{ss}}{1 + \pi_{ss}}, 1 + \pi_{ss}\right)$$

$$\frac{M}{P^*} = L\left(\frac{1 + i_{ss}}{1 + \pi_{ss}}, 1 + \pi_{ss}\right)$$

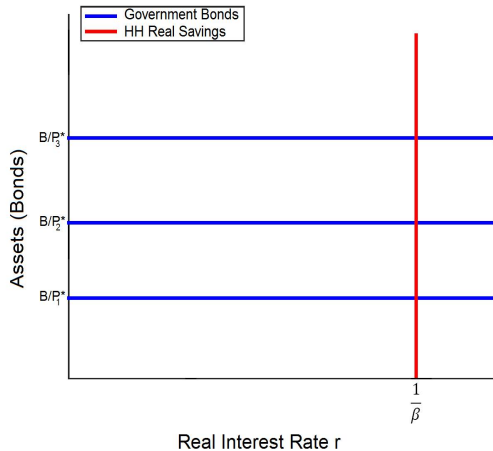
$$\frac{B}{P^*} = S\left(\frac{1 + i_{ss}}{1 + \pi_{ss}}, 1 + \pi_{ss}\right) + L\left(\frac{1 + i_{ss}}{1 + \pi_{ss}}, 1 + \pi_{ss}\right)$$

$$1 + \pi_{ss} = 1 + \frac{B' - B}{B} = 1 + \frac{M' - M}{M}$$

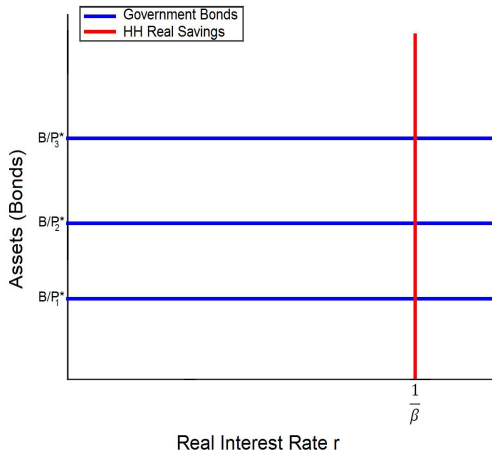
Central bank provides

$$M = P^* L\left(\frac{1 + i_{ss}}{1 + \pi_{ss}}, 1 + \pi_{ss}\right)$$

STEADY STATE PRICE LEVEL: COMPLETE MARKETS



STEADY STATE PRICE LEVEL: WHY TANK DOES NOT DELIVER



Real Interest Rate:

$$(1 + r) = \frac{1+i}{1+\pi}$$

Monetary Policy:

Sets $1 + i$

Fiscal Policy:

$$\pi = \frac{B' - B}{B} = \frac{G' - G}{G} = \frac{T' - T}{T}$$

i : nominal interest rate

r : real interest rate

π : inflation rate

B : nominal bonds

G : nominal government spending

T : nominal tax revenue